

HW #96

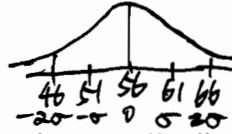
MR12 Worksheet (Normal Distribution) (Key)

P.1

1. The mean of a normally distributed set of data is 56, and the standard deviation is 5. In which interval do approximately 95.4% of all cases lie?

- (1) 46-56 (2) 51-61 (3) 46-66 (4) 56-71

since 95.4% is about 2σ ,



46-66

2. The amount of juice dispensed from a machine is normally distributed with a mean of 10.50 ounces and a standard deviation of 0.75 ounce. Which interval represents the amount of juice dispensed about 68.2% of the time?

- (1) 9.00-12.00 (2) 9.75-11.25 (3) 9.75-10.50 (4) 10.50-11.25

68.2% is 1σ , so, $10.5 + 0.75 = 11.25$
 $10.5 - 0.75 = 9.75$, $9.75 - 11.25$

3. In a New York City high school, a survey revealed the mean amount of cola consumed each week was 12 bottles and the standard deviation was 2.8 bottles. Assuming the survey represents a normal distribution, how many bottles of cola per week will approximately 68.2% of the students drink?

- (1) 6.4 to 12 (2) 9.2 to 14.8 (3) 6.4 to 17.6 (4) 12 to 20.4

68.2% again, is 1σ , $12 + 2.8 = 14.8$
 $12 - 2.8 = 9.2$ 9.2 - 14.8

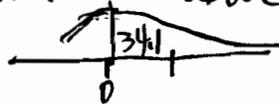
4. On a standardized test, the distribution of scores is normal, the mean of the scores is 75, and the standard deviation is 5.8. If a student scored 83, the student's score ranks

- (1) below the 75th percentile (2) between the 75th percentile and the 84th percentile
 (3) between the 84th percentile and the 97th percentile (4) above the 97th percentile

$83 - 75 = 8$

$8 \div 5.8 = 1.38\sigma$

1.38 σ is above 84% ($50 + 34$) = 84%

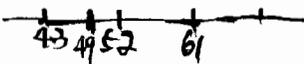


so (3) is the best choice

5. Scores on PSAT range from 20 to 80. For a certain population of students, the mean is 52 and the standard deviation is 9.

How many standard deviations from the mean for a) 49 b) 56 c) 64 d) 65?

- a) $-1/3$ b) $4/9$ c) $4/3$ d) $13/9$

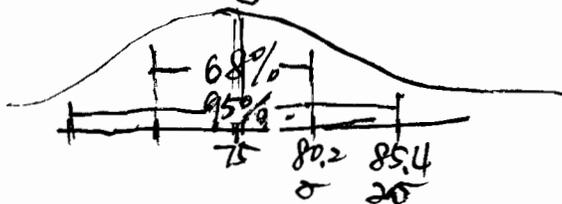


a) $49 - 52 = -3$
 $\frac{-3}{9} = -\frac{1}{3}\sigma$

b) $56 - 52 = 4$ c) $64 - 52 = 12$ d) $65 - 52 = 13$
 $4 \div 9 = \frac{4}{9}\sigma$ $\frac{12}{9} = \frac{4}{3} = 1\frac{1}{3}\sigma$ $\frac{13}{9} = 1\frac{4}{9}\sigma$

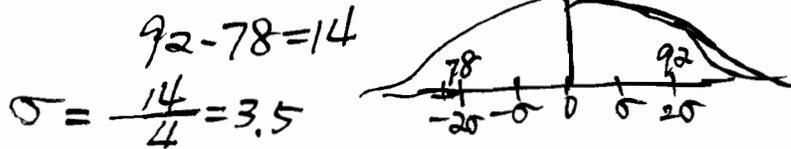
6. In a standardized test with a normal distribution of scores, the mean was 75 and the standard deviation was 5.2. Which score could be expected to occur less than 5% of the time?

- a) 95 b) 85 c) 75 d) 65



$75 + 2(5.2) = 85.2$

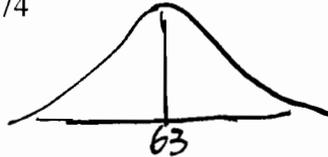
7. In the diagram, the shaded region represents approximately 95% of the scores on a standardized test. If these scores ranged from 78 to 92, what would be the standard deviation?



8. In a standardized test with a normal distribution of scores, the mean is 63 and the standard deviation is 5. Which score could be expected to occur most often?

- a) 45 b) 55 c) 63 d) 74

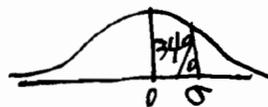
the closer to the mean
the higher frequency it gets.



9. Battery lifetime is normally distributed for large samples. The mean lifetime is 500 days and the standard deviation is 61 days. Approximately what percent of batteries have lifetimes longer than 561 days?

- (1) 16% (2) 68% (3) 34% (4) 84%

1σ

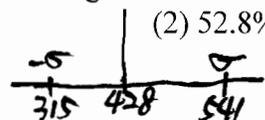


$50\% + 34\% = 84\%$ $100 - 84 = 16\%$

10. The national mean for verbal scores on an exam was 428 and the standard deviation was 113. Approximately what percent of those taking this test had verbal scores between 315 and 541?

- (1) 68.2% (2) 52.8% (3) 38.2% (4) 26.4%

$428 + 113 = 541$
 $428 - 113 = 315$



exactly within 1σ.

11. Twenty high school students took an examination and received the following scores:

70, 60, 75, 68, 85, 86, 78, 72, 82, 88, 88, 73, 74, 79, 86, 82, 90, 92, 93, 73

Determine what percent of the students scored within one standard deviation of the mean. Do the results of the examination approximate a normal distribution? Justify your answer.

mean = 79.7, standard deviation = 8.7
the range of one standard deviation is from 71 to 88.4
 $14/20 = 70\%$

Yes, since 68% of scores fall within one standard deviation from the mean for a normal distribution.

12. Mrs. Ramírez is a real estate broker. Last month, the sale prices of homes in her area approximated a normal distribution with a mean of \$150,000 and a standard deviation of \$25,000.

A house had a sale price of \$175,000. What is the percentile rank of its sale price, to the nearest whole number? Explain what that percentile means.

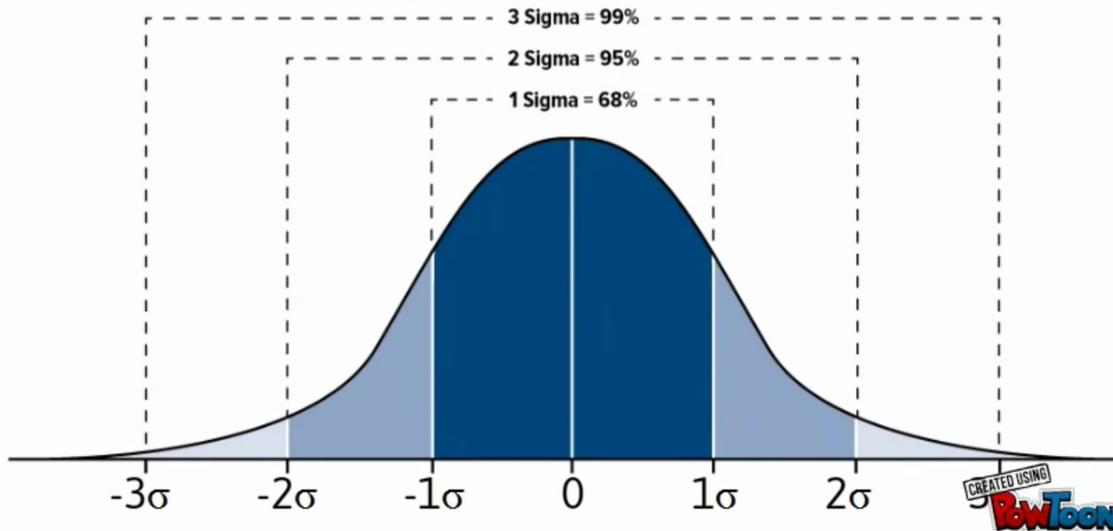
Mrs. Ramírez told a customer that most of the houses sold last month had selling prices between \$125,000 and \$175,000. Explain why she is correct.

\$175,000 is on the one standard deviation, therefore is on 84% percentile.

There is 84% percent of houses sold for less than or equal to \$175,000

\$125,000 to \$175,000 is the range of one standard deviation which has 68% of probability to be occurred.

68-95-99.7 RULE



7. In the diagram, the shaded region represents approximately 95% of the scores on a standardized test. If these scores ranged from 78 to 92, what would be the standard deviation?
8. In a standardized test with a normal distribution of scores, the mean is 63 and the standard deviation is 5. Which score could be expected to occur most often?
a) 45 b) 55 c) 65 d) 74
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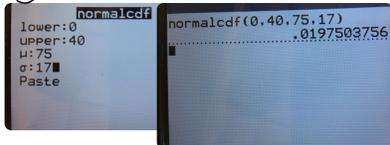
More Exercises

1. The angiogram is a standard diagnostic test used in clinical medicine to detect stroke in patients. This test has some risks for the patient, and several noninvasive techniques have been developed that are hoped to be as effective as the angiogram. One such method utilizes that measurement of cerebral blood flow (CBF) in the brain, since stroke patients tend to have lower levels of CBF than normal. Among healthy people, CBF is normally distributed with mean 75 and standard deviation 17. Patients are classified as being at risk for stroke if their CBF is below 40. What proportion of normal patients will be mistakenly classified as being at risk for stroke?
2. Maple tree diameters in a forest area are normally distributed with mean 10 inches and standard deviation 2.2 inches. Find the proportion of trees having a diameter greater than 15 inches.
3. Our subjects are 35-44-year-old males whose blood pressures are normally distributed with mean 80 and standard deviation 12. A borderline hypertensive is defined as a person whose diastolic blood pressure is between 90 and 95 mm Hg inclusive; what proportion of subjects are borderline hypertensive? A hypertensive is a person whose diastolic blood pressure is above 95 mm Hg; what proportion of subjects are hypertensive?
4. White blood cell (WBC) count per cubic millimeter of whole blood has approximately the Normal distribution with mean 7500 and standard deviation 1750. The lowest 2% of all WBC counts are defined to be probable risks. How low must one's WBC count be to fall in the at-risk group?
5. The resting heart rate for healthy adult horses averages 46 beats per minute with a standard deviation of 8 beats per minute. A horse whose resting heart rate is in the upper 10% of the distribution of heart rates may have a secondary infection or illness that needs to be treated. How fast must a healthy horse's heart be beating to fall into this at-risk group?

Solutions

1. 0.0198
2. 0.0115
3. 0.0967, 0.1056
4. 3905.94 (3906 is fine)
5. 56.25

①



②



③A



③B



④



⑤

