

CHAPTER 9

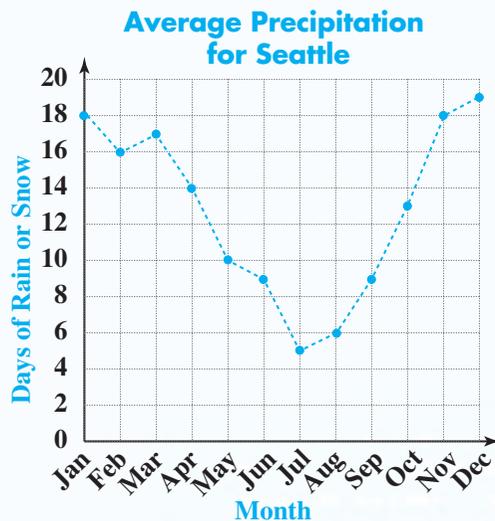
Interpreting Graphs

Real-Life Math

Rain, Rain, Go Away When the student council at Cedar Lane Middle School in Seattle, Washington, started planning its annual car wash benefit, the members needed to agree on a date. They knew that the car wash would be less successful on a rainy day, so they decided to choose a day in a month when it doesn't usually rain very much. They consulted the graph at the right to help them make their decision. Which month do you think would be the best choice?

The students also created other graphs to help them plan for and track the results of their car wash.

Think About It Can you think of what kinds of information the students may have used to make their other graphs?



Family Letter

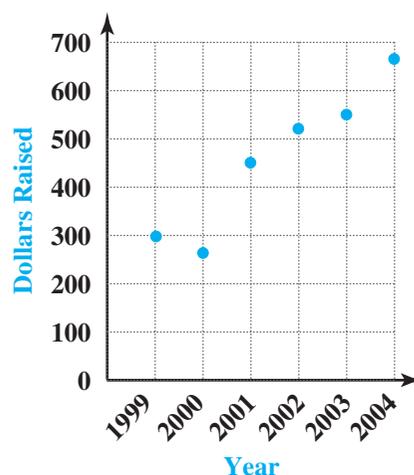
Dear Student and Family Members,

In our next chapter, we will be interpreting graphs and understanding the stories behind them. For example, the graph at the right shows the amount of money raised at car washes held by the student council.

Here are some ways we can interpret the graph to learn more of the story:

- About how much more money was raised in 2004 than in 2003?
- Between which two years was there a decrease in the number of dollars that were raised?

Money Raised at Car Washes



Many graphs represent mathematical relationships, which can also be represented by equations or formulas. We will learn about the shapes of graphs that represent repeating, linear, and quadratic relationships as well as exponential relationships, which show situations where a quantity is repeatedly multiplied by a number greater than 1.

Vocabulary Along the way, we'll learn about two new vocabulary terms:

line graph

multiplicative inverse

What can you do at home?

You can help your student read the stories behind graphs by discussing graphs you find in magazines, newspapers, web sites, or advertisements. Try to find the story behind the graph by asking these questions:

- What is the graph about?
- What does each point represent?
- Is there a comparison between items in the graph?
- Can you use the graph to predict something that is not explicitly shown on the graph?

Finding the story behind the graph can be an enjoyable and imaginative experience that you can share with your student. Have fun!



9.1

Graphing Change over Time

The shape of a graph can tell you a lot about the relationship between two variables. When the variable on the horizontal axis is time, the graph shows how the variable on the vertical axis changes as time passes.

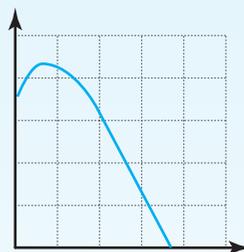
Think & Discuss

Kate's mother is a health care worker at a community clinic. As part of her job, she sometimes creates and analyzes graphs. Many of the graphs she makes show how variables change over time.

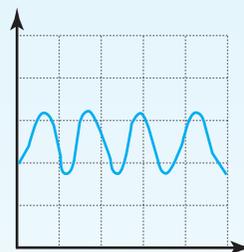


Match each situation below to the graph you think is most likely to describe how the variable in the situation changes. Explain your choices.

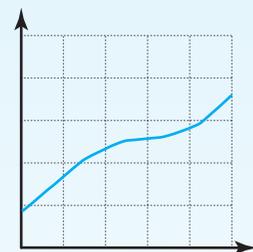
1. a newborn baby's weight during the first few months of life
2. the number of spots on a child with chicken pox from when Kate's mother first sees him until he is well again
3. the monthly number of flu cases reported over several years



Graph A



Graph B



Graph C

Investigation

1

Growing Up

Each time one of his daughters celebrates a birthday, Mr. Fernandez measures her height and records it on a chart.

Age (years)	Height (inches)		
	Maria	Luisa	Rosi
3			$36\frac{1}{2}$
4		37	$37\frac{1}{2}$
5		$37\frac{3}{4}$	$40\frac{1}{2}$
6	45	42	$43\frac{1}{2}$
7	$46\frac{1}{2}$	44	$44\frac{1}{2}$
8	$48\frac{1}{4}$	$45\frac{1}{4}$	$46\frac{1}{2}$
9	$50\frac{1}{2}$	47	$47\frac{1}{2}$
10	52	$47\frac{3}{4}$	
11	54		
12	$57\frac{1}{2}$		



Problem Set A

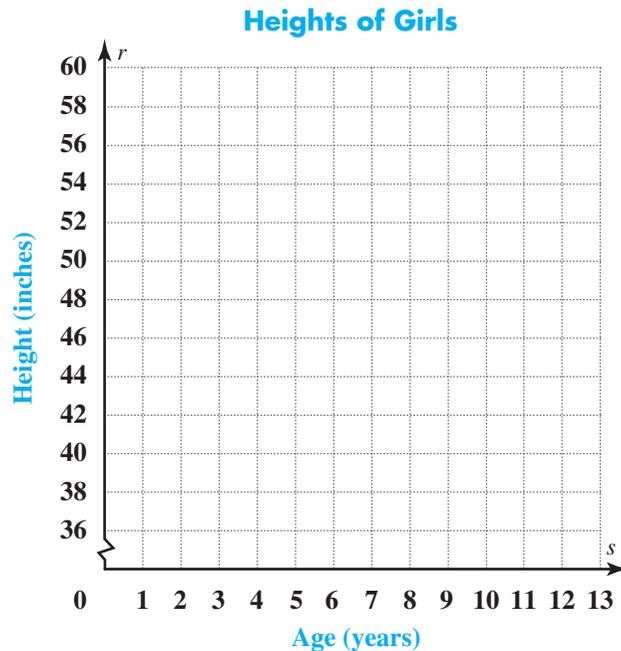
Use the information from Mr. Fernandez's chart to answer these questions.

1. Why might some of the height values be missing?
2. How tall was each girl at age 6?
3. Put the girls' names in order, from the girl who was shortest at age 6 to the girl who was tallest.
4. Put the girls' names in order, from the girl who was shortest at age 7 to the girl who was tallest.
5. How much did each girl grow between her sixth and seventh birthdays?
6. Put the girls' names in order, from the girl who grew least between 6 years and 7 years to the girl who grew most.
7. Was the girl who grew most between 6 and 7 years the tallest at 7 years? Explain.

Graphing the data in Mr. Fernandez’s growth chart can give you an overall picture of how the girls’ heights changed over the years.

Problem Set B

1. On a grid like the one below, plot points to show how tall Maria was at each birthday.



Of course, Maria’s height didn’t suddenly change on each of her birthdays! The chart and graph show how tall Maria was on each birthday but not how tall she was in between birthdays.

2. You can use your graph to estimate Maria’s height at times between her birthdays. Connect the points on your graph to get an idea of how Maria grew during each year.

VOCABULARY

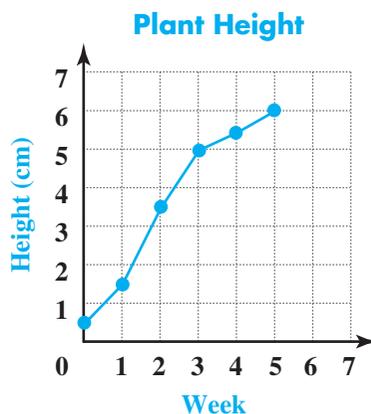
line graph

When the points on a graph are connected by line segments, the graph is called a **line graph**.

3. If Mr. Fernandez had taken measures every month between Maria's birthdays, would the data points between any two birthdays be likely to fall on a straight line? Why or why not?
4. On the same grid you used for Maria, draw line plots for the heights of Luisa and Rosi. You might want to use a different color for each girl.
5. Use your graph to predict Rosi's height at 10 years of age.
6. How can you tell from the graphs which girl grew most between ages 6 and 7?
7. Between which two birthdays did Luisa grow most? Between which two birthdays did she grow least?
8. Describe an easy way to use the graph to find the year in which Rosi grew most.
9. Think about what you found in Problems 6–8. What can you conclude about the shapes of the graphs and the rates at which the girls grow?

Share & Summarize

The graph shows how a plant grew over time, beginning when it first appeared above the ground.



1. How tall was the plant after 1 week?
2. When was the growth rate fastest, and when was it slowest? Explain.
3. Estimate how tall the plant was after 10 days. How did you find your answer?

Investigation 2 Filling It Up

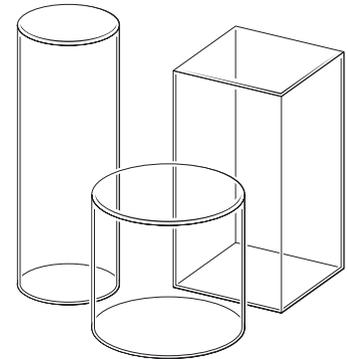
In this investigation, you will examine how the water level in containers of various shapes changes as water is poured into them and draw graphs of the data you collect.

MATERIALS

- cup or scoop
- large container of water tinted with food dye
- three transparent jars of different sizes and with straight, vertical sides
- 3 pencils or pens of different colors
- a marker
- metric ruler
- graph paper

Problem Set C

One member of your group should get three different jars for the group. Suppose you use a scoop to pour the same amount of water into each jar. You mark the water levels on the jars, add another scoop of water to each jar, and mark the levels again. You continue scooping and marking until one jar is nearly full.



1. Sketch graphs to show how you think the water level will rise in each jar as you pour water into it. Put the graphs on the same set of axes, and use a different color for each graph. Put height on the vertical axis and number of scoops on the horizontal axis.

Now try it. One person should pour a scoop of water into each jar. Another should mark the water's height on the side of the jar after each scoopful. Put the same number of scoopfuls in each jar, and continue until one jar is nearly full.

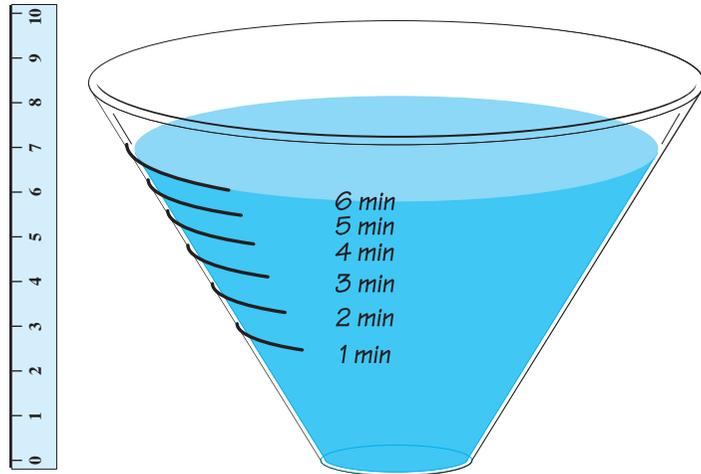
2. Measure the heights of the marks you made on each jar. Record the information in a table like this one.

Scoopfuls	Jar 1	Jar 2	Jar 3
0	0 cm		
1			
2			
3			
4			
5			

3. On a new set of axes, draw three graphs, one for each jar, that show the actual heights of the water plotted against the number of scoops. Use the same colors for each jar that you used in Problem 1.
4. Examine the patterns made by your graphs. What do you notice?
5. Compare these graphs with the predictions you drew in Problem 1. Write a paragraph comparing your predictions with the actual results.

Problem Set D

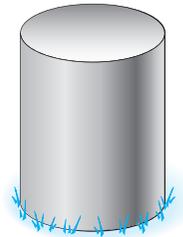
Eva and Tayshaun allowed water from a leaking faucet to fall into the cup shown below. They marked the height of the water on the cup every minute. At the end of 6 minutes, the cup looked like this.



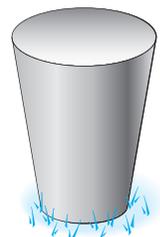
1. Why do the marks get closer together as time passes?
2. Use the ruler at the side of the cup to find the height of the water at each minute. Draw a line graph of the data, and describe its shape.
3. Have you noticed that often when people fill bottles and vases with a narrow top, the water suddenly gushes out the top? Why do you think this happens?

Share & Summarize

1. Ramona owns a cabin high in the mountains. Her main water source is a tank shaped like a cylinder. To fill the tank, Ramona hires a small tanker truck. The tanker's pump sends a steady stream of water into the tank. Make a rough sketch of a graph that shows how the height of the water in the tank will change as it fills.



2. George owns the cabin nearest to Ramona's. He also uses a water tank, but his tank has sides that slant outward slightly. Make a rough sketch of a graph that shows how the height of the water in George's tank will change as the tanker truck fills it with a steady stream of water.

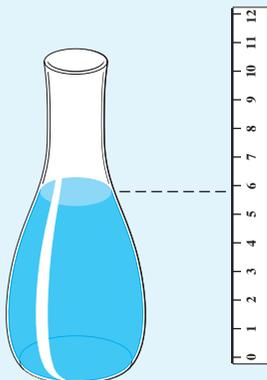


Lab Investigation

Filling Odd-shaped Containers

MATERIALS

- cup or scoop
- large container of water tinted with food dye
- three transparent containers of odd shapes
- a marker
- metric ruler
- graph paper



In Investigation 2, you discovered what happens to the height of the water as it fills containers with straight and slanted sides. Some containers don't have such simple shapes.

Make a Prediction

With your group, get three containers with different shapes. As you did in Investigation 2, imagine filling the containers with water, one scoop at a time.

1. For each container, sketch a rough graph that shows how you expect the height of the water to change as you fill the container. Discuss your graphs with your group, and redraw your sketches if necessary. Explain why you think the graphs will look like your sketches.

Try It Out

Now try the experiment. One person should pour a scoop of colored water into each container. Another should mark the height of the water on the side of the container after each scoopful. Continue with each container until they are all filled.

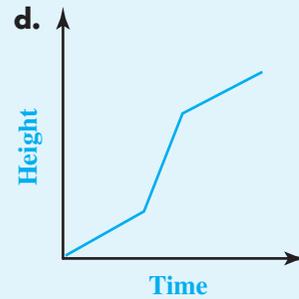
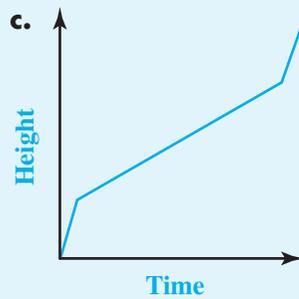
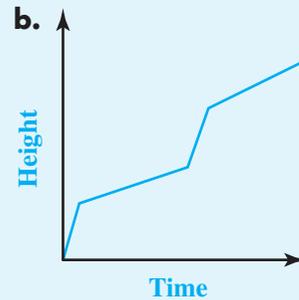
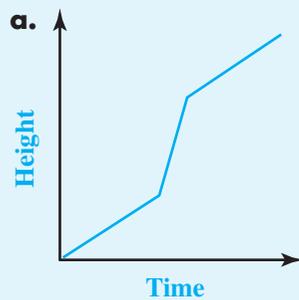
2. Measure the heights of the marks you made on each container. Be sure to measure the vertical height, as shown at left. Record the information in a table.
3. Use the data from your table to draw a graph for each container.
4. Were your sketches roughly the right shapes? If they weren't, try to explain why your predictions weren't accurate.

Design Your Own Container

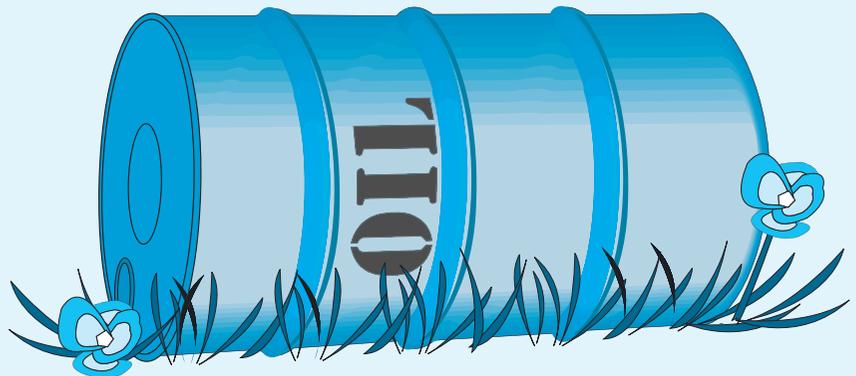
5. Design an odd-shaped container and draw it.
6. On a different sheet of paper, sketch a graph that shows how you expect the water height to change as you fill your container.
7. Trade graphs with another member of your group. Look at the graph you have been given, and try to imagine what shape container might give that graph. Make a rough sketch of the container you imagine, and then return it to your classmate.
8. Compare your container design to the container your classmate drew. Do the containers look alike? If not, could your graph match both containers?

What Did You Learn?

9. Draw the container that might go with each graph. All four containers hold the same amount of liquid, and it is poured at a constant rate.



10. Tanks that hold oil for heating houses are often cylindrical. However, they are usually installed so that the circular bases are on the sides rather than at the top and bottom.

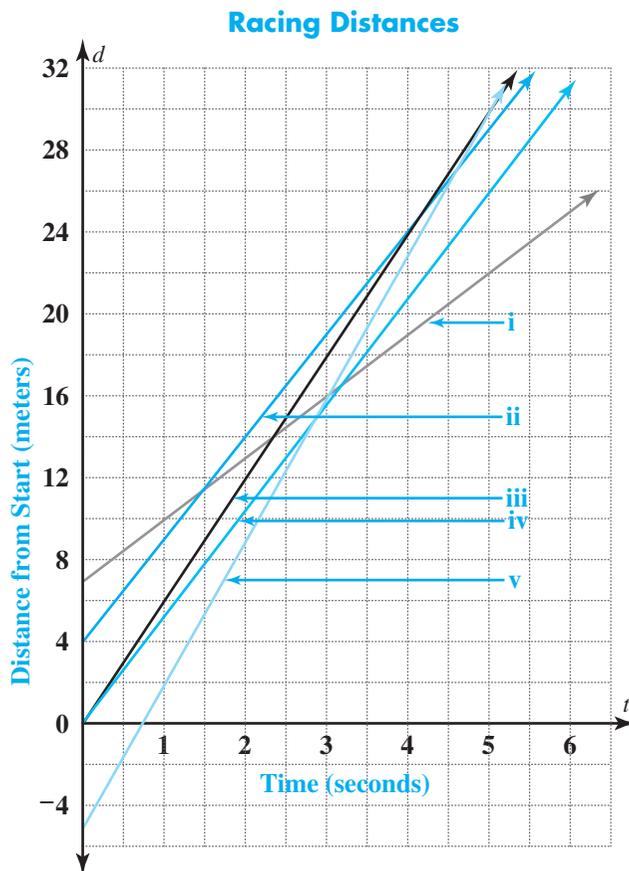


The tank's fuel gauge uses the height of the oil inside the tank to show how full the tank is. Oil tanks are not filled completely to allow for some expansion if the temperature changes. However, if the tank were filled completely, the height of the oil in a full tank would be equal to the diameter of the tank.

- a. Sketch a graph that shows how the height of the oil changes as you fill the tank.
- b. **Challenge** When the tank is $\frac{1}{4}$ full, is the height of the oil $\frac{1}{4}$ of the tank's diameter? Explain.

Investigation 3 Walking About

Graphs are often used to show where an object—a person, an ant, a ball, a car, a planet, or anything else that moves—is located at various times. In Chapter 5, for example, you considered a race between five brothers. The graphs of their distances from the starting line are shown below.



Think & Discuss

A graph's shape illustrates, or gives information, about a situation.

- The graphs all have different slopes. What does that tell you about the brothers in the race?
- To give the slower boys a chance to win, the brothers started at different places. How is this shown in the graph? Explain.
- Suppose the brothers' dog, King, was waiting at the finish line. When the boys started running, King ran toward them. After a couple of seconds, he turned around and ran back to the finish line. Give a rough description of what King's graph would look like.

Problem Set E

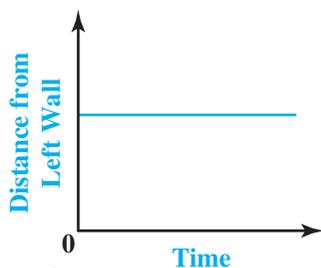
In the following problems, a person walks from one side of a room to the other. You can match graphs with different walks by thinking about the situation.



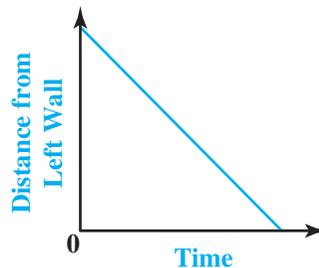
Discuss each problem below with your partner until you are sure of your answers and feel you could convince someone else you are correct.

- Zach walks slowly and steadily across the front of his classroom, from the left wall to the right wall. Which graph best shows his distance from the left wall at any time? Explain your reasoning to your partner.

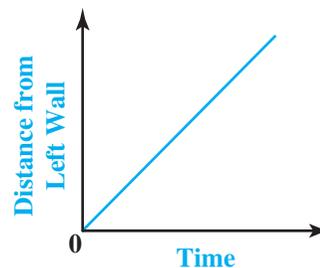
Graph a



Graph b

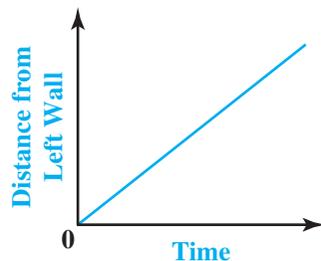


Graph c

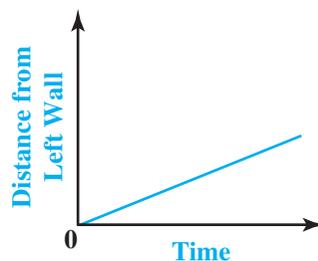


- Each graph below uses the same scales on the axes. Which shows the fastest walk from left to right, and which shows the slowest? Explain your reasoning to your partner.

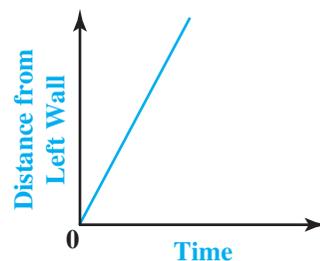
Graph d



Graph e



Graph f

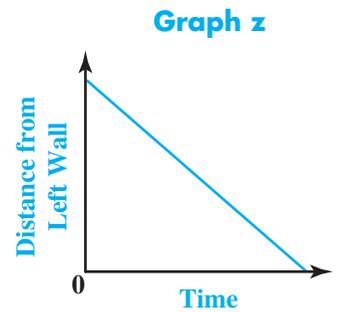
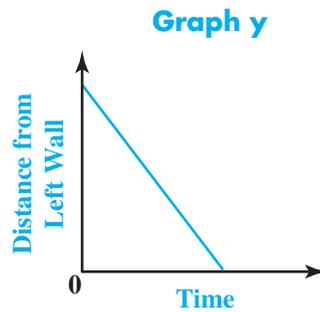
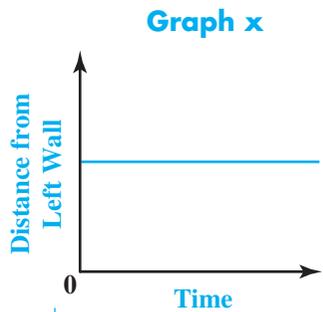
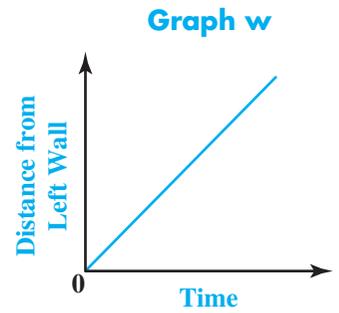
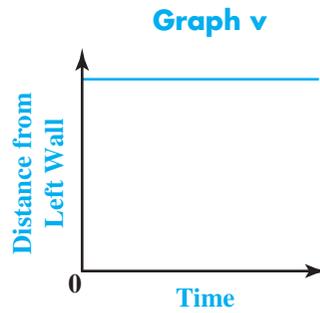
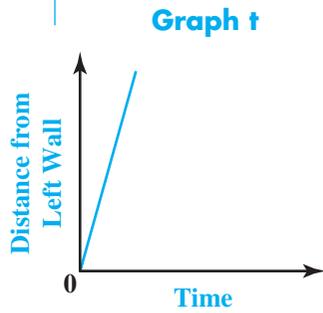


- Suppose Jin Lee walks in the opposite direction, from the right wall to the left wall. Sketch a graph of Jin Lee's distance from the *left* wall over time.

4. The graphs below all use the same scale.

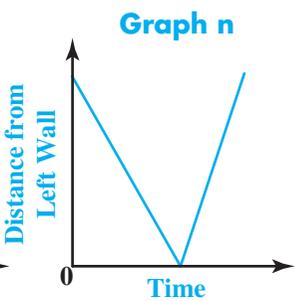
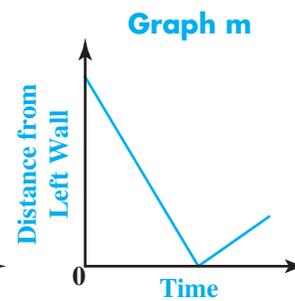
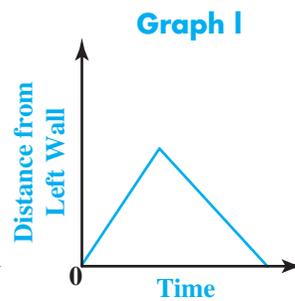
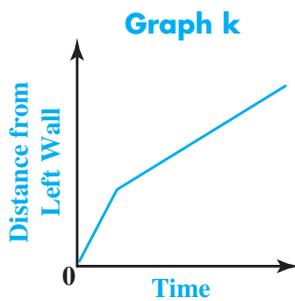
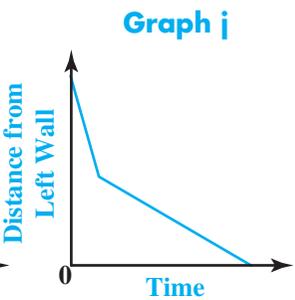
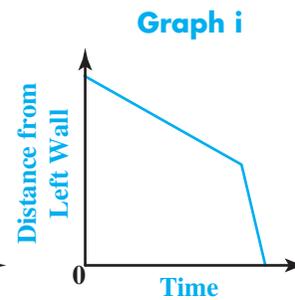
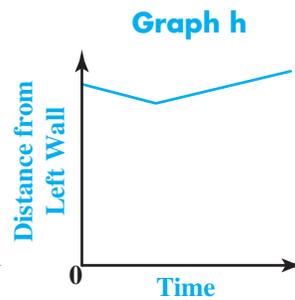
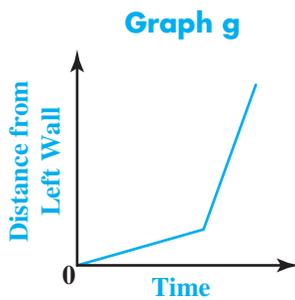
a. Choose the graph that best represents the following:

- i. walking slowly from left to right
- ii. walking slowly from right to left
- iii. walking quickly from left to right
- iv. walking quickly from right to left
- v. standing still in the center of the room



b. There is one extra graph. Describe the walk it shows.

5. Look at the eight graphs below.



- a.** Choose the graph that best shows each of these walks:
- i.** slow to the right, then fast to the right
 - ii.** fast to the right, then slow to the right
 - iii.** slow to the left, then fast to the left
 - iv.** fast to the left, then slow to the left
 - v.** slow to the left, then slow to the right
 - vi.** fast to the right, then fast to the left
 - vii.** fast to the left, then fast to the right
- b.** There is one extra graph. Describe the walk it shows.

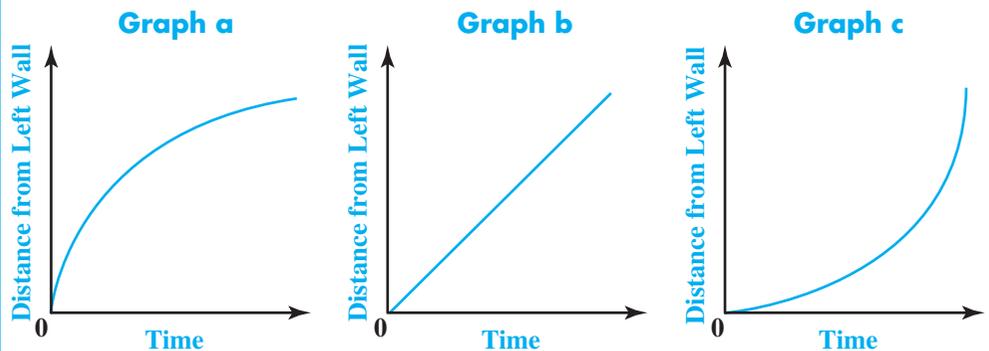
Problem Set F

Sophie starts to walk slowly from the left wall of a large hall and moves faster as she walks to the right.

- 1.** Suppose you mark the floor at fixed intervals of time—for example, every 2 seconds—to show where Sophie was as she walked across the room.

Which of the following statements would best describe the marks? If necessary, you and your partner can experiment to discover what happens.

- a.** The marks will be equally spaced across the room.
 - b.** The marks will get further apart from left to right.
 - c.** The marks will get closer together from left to right.
- 2.** Which of these graphs best describes Sophie's walk? Explain how the graph links to the description you chose in Problem 1.

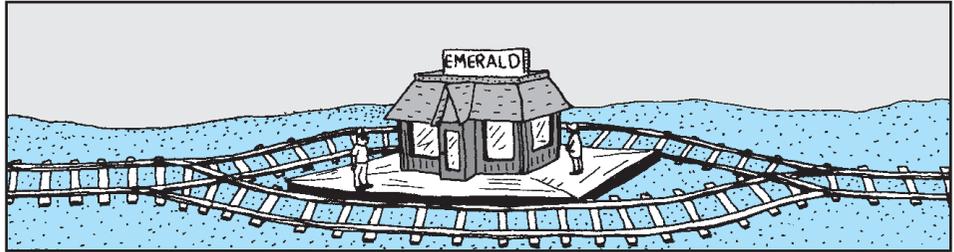


- 3.** What kinds of walks are shown in the other graphs of Problem 2?

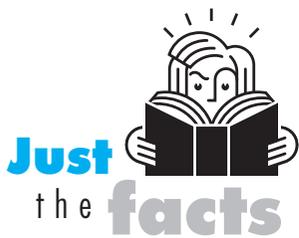
Problem Set **G**

Many cities around the world use historical trains for tourist trips. The famous steam train *Puffing Billy* in the hills near Melbourne, Australia, is really a set of several small trains. About three trains run on any day, taking people from Belgrave to Lakeside and back.

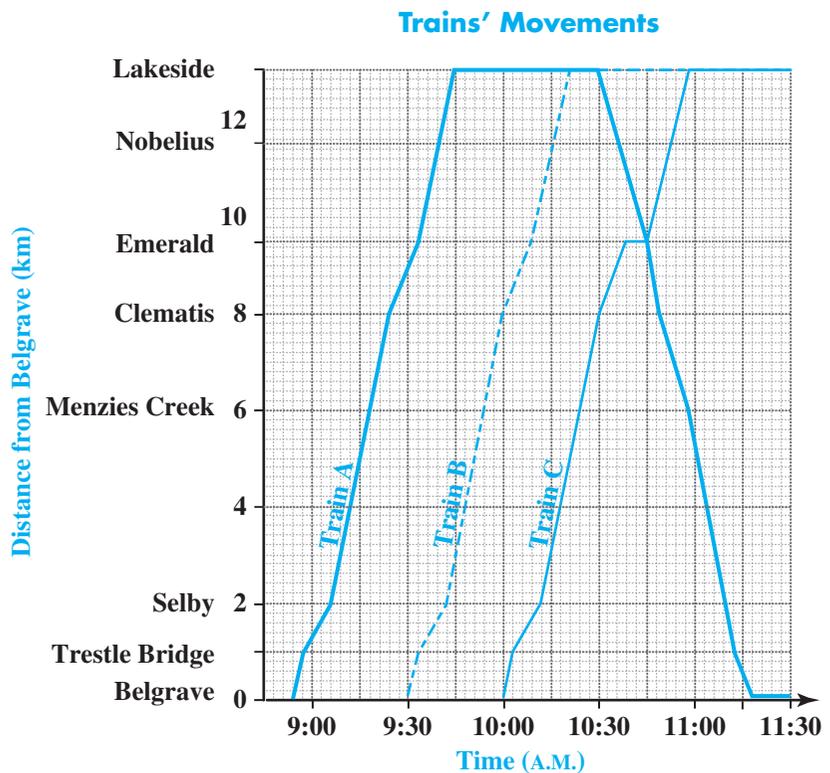
The line is mainly a single track, so when trains from each direction meet, one has to wait for the other to pass. Of the eight stations along the route, Menzies Creek and Emerald are the only ones with double tracks.



One day Train A left the Belgrave station at 8:54 A.M., Train B at 9:30 A.M., and Train C at 10:00 A.M. Train C met Train A as Train A was making the return trip. The graphs show the trains' movements up to 11:30 A.M. The names of the stations and their distances from Belgrave are shown on the vertical axis.



You may not have a good sense of speeds in kilometers per hour. Trains take 8 minutes to travel the 1 kilometer from Trestle Bridge to Selby—a speed of 7.5 kph. An average walking speed for a person is about 5 kph.



1. When did Train A arrive in Lakeside?
2. For how long did Train A stop at Lakeside?

- The section from Trestle Bridge to Selby is the steepest climb. How do the graphs show this? Explain how this relates to the speed of the trains.
- Which train took the longest to get from Belgrave to Lakeside? Why do you think it needed more time?
- Along which sections did Train A reach its greatest speed on the return trip to Belgrave?
- It is downhill from Emerald to Clematis. How does the graph show that Clematis to Emerald is a steep climb?
- At 11:30 A.M., where was each of the three trains?

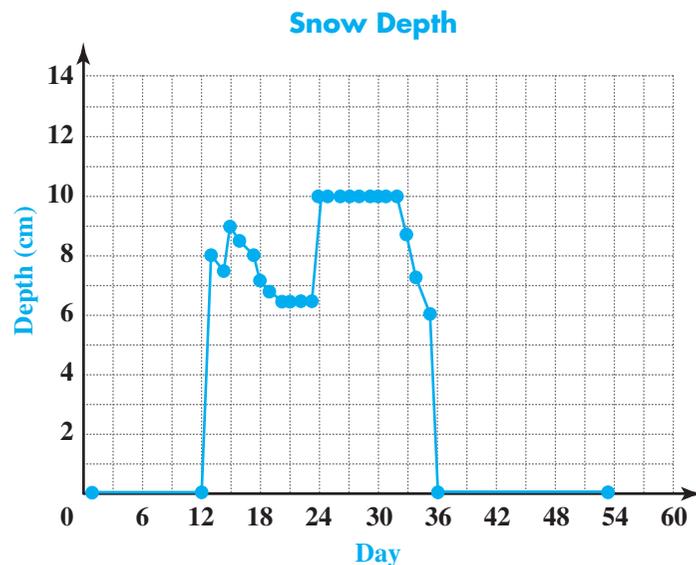
MATERIALS

graph paper



Share & Summarize

One winter, Malik conducted a weather experiment. He stuck a meterstick upright in his backyard on December 1. Every morning when there was snow on the ground, he measured the depth of the snow. He made this graph from his data, counting December 1 as Day 1.

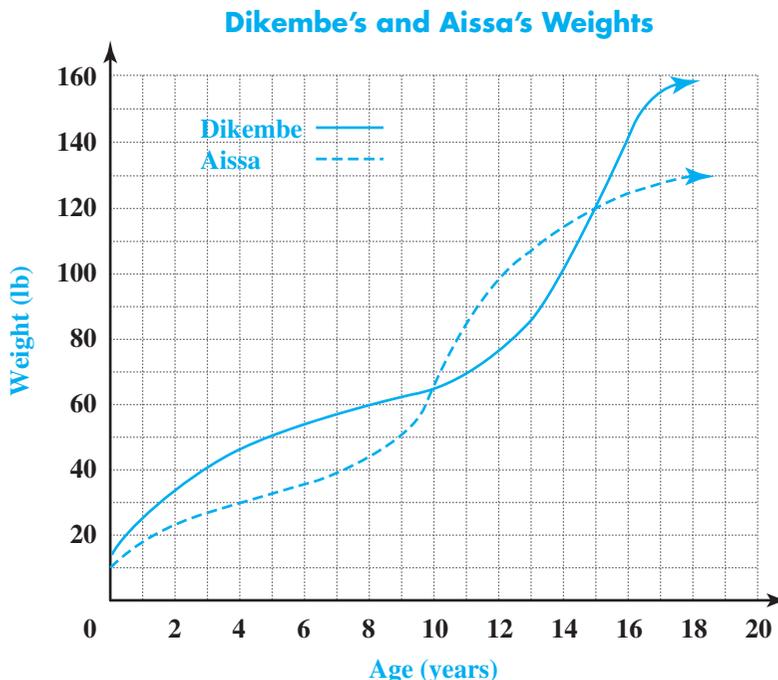


- When did the first snow fall? How much fell?
- During a sudden warm spell, rain washed away all the snow left from the last storm. How is this event shown on the graph? When was the rainfall?
- That winter, school was canceled because of snow only once, on January 23. Make a rough copy of the graph, and add a section showing what it might look like by January 24.

On Your Own Exercises

Practice & Apply

1. Dikembe and Aissa are twins. The graphs show their weights during the first 20 years of their lives.



- From the graphs, how much weight did each twin gain between 10 and 18 years of age?
- Between what ages did Dikembe weigh more than Aissa did at those same ages? Explain how you know from the graph.
- At what ages did the twins weigh the same?
- When was Aissa gaining weight most rapidly? Explain how you know from her graph.
- Estimate how much weight per year Aissa was gaining at that time.
- When was Dikembe's rate of weight gain greatest? Estimate his rate of weight change at that time.
- Who was putting on weight faster at age 16? How do you know?



2. The tables give average heights of girls and boys for different ages.

Girls

Age (yr)	2	4	6	8	10	12	14	16
Average Height (in.)	35.6	40.1	45.3	50.0	53.8	58.5	63.1	64.0

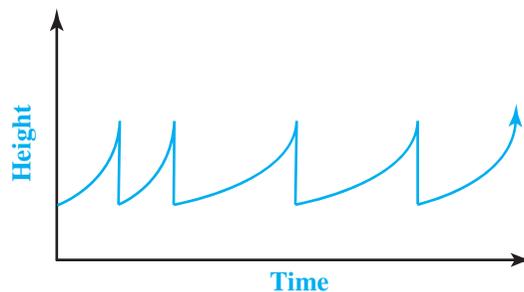
Boys

Age (yr)	2	4	6	8	10	12	14	16
Average Height (in.)	34.2	40.6	45.6	50.0	54.2	59.0	64.2	68.4

Source: National Center for Health Statistics

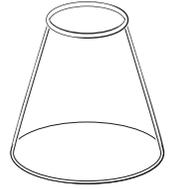
By graphing the data, you can get an idea of when young people usually grow the most and the least. You can easily compare the information for girls and boys.

- On a grid, plot girls' average heights at different ages. Label the vertical axis from 30 in. to 70 in. Should you connect the points? Explain.
 - In another color, plot boys' average heights at different ages. If it makes sense, connect the points.
 - Use your graph to estimate the average height of girls and of boys at 15 years of age.
 - From your graph, decide at what ages boys and girls are, on average, the same height and when they are not.
 - Write a brief paragraph describing the relationship between age and average height for boys and for girls.
3. The graph shows the height of grass as it grows over time.



- When the graph is steep and increasing, what is happening to the grass?
- When the graph is not so steep, what is happening to the grass?
- The vertical line segments show the most dramatic change. What might these show?

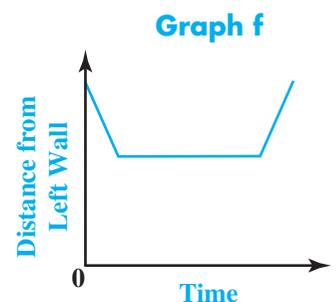
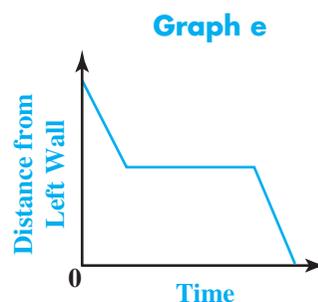
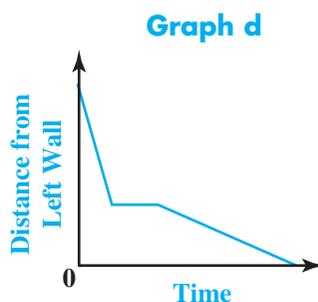
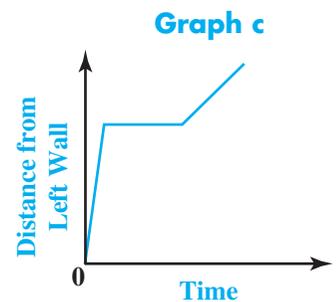
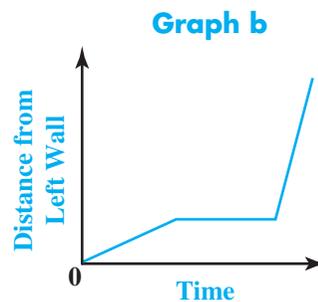
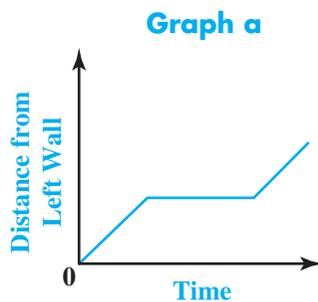
4. Many science laboratories have beakers with sides that slant inward. Sketch a graph that shows how the water level in the beaker would change if water is poured in at a constant rate.

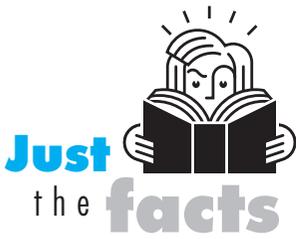


5. Ramona's cabin has a water tank shaped like a right cylinder. George owns the cabin nearest to Ramona's, and his water tank has sides that slant outward slightly. (See the drawings on page 607.) When they close up their places at the beginning of the winter, they empty their tanks so there isn't any water in them to freeze during the winter.

Assuming the water drains from the tanks at a constant rate, sketch graphs that show how the water height in each tank will change as the tanks drain.

6. Match these descriptions of a person's walk across a room with the graph that best shows the walker's distance from the left wall.
- i. slow to the right, stop, slow to the right
 - ii. fast to the right, stop, slow to the right
 - iii. fast to the left, stop, fast to the right
 - iv. slow to the right, stop, fast to the right
 - v. fast to the left, stop, fast to the left
 - vi. fast to the left, stop, slow to the left



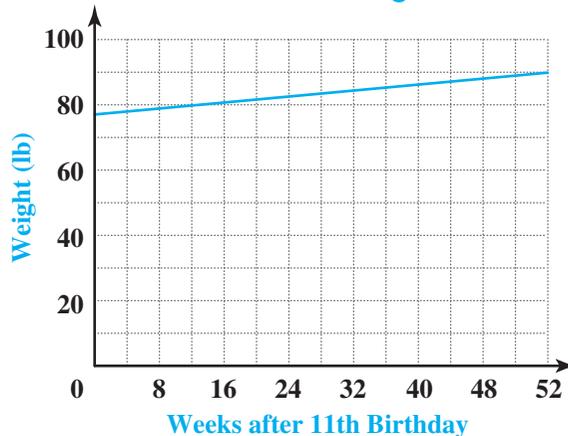


Forrest Gump, made in 1994, is one of the top 10 money-making films of all time, grossing more than \$329 million in ticket sales by 1999.

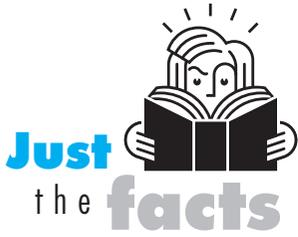
Connect & Extend

7. In the movie *Forrest Gump*, the hero decides to run across America.
 - a. If Forrest runs an average of 5 miles per hour for 10 hours each day, how many miles per day will he cover?
 - b. New York to San Francisco is approximately 4,000 miles. If Forrest maintains his pace, how many days will this journey take?
 - c. If Forrest rests for 10 days at the halfway mark, how long would the entire journey take?
 - d. Draw a graph of the journey, including the halfway rest. Put time (days) on the horizontal axis and miles on the vertical axis.
 - e. On the same graph, draw a line for someone who gave Forrest 20 days' head start and arrived in San Francisco at the same time. What must this person's average speed be if he or she also ran for 10 hours per day?
8. The graph shows approximately how Charles' weight changed the year he was 11.

Charles' Weight



- a. What did Charles weigh on his 11th birthday? Explain how you know this from the graph.
- b. Use data from the graph to calculate Charles' average weight gain each week.
- c. Write a rule in words and in symbols to describe the relationship between time after Charles' birthday t in weeks and weight w in pounds.
- d. Assuming Charles continues to grow at the same rate, how old would you expect him to be when he weighs 100 lb? 120 lb?
- e. If Charles continues to grow at the same rate, what will his weight be when he is 15 years old? 20 years old?



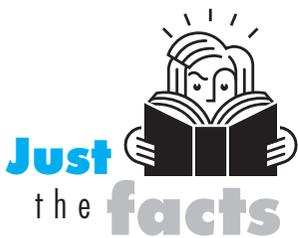
U.S. censuses conducted before 1950 did not count people living in what are now the states of Alaska and Hawaii.

9. Social Studies The population of the United States has grown steadily over the years. The table shows the population at various intervals. The data are from the U.S. Census, which is taken every 10 years.

U.S. Population

Year	Population (millions)	Percent Increase	Year	Population (millions)	Percent Increase
1790	3.9	—	1900	76.2	
1800	5.3		1910	92.2	
1810	7.2		1920	106.0	
1820	9.6		1930	123.2	
1830	12.9		1940	133.2	
1840	17.1		1950	151.3	
1850	23.2		1960	179.3	
1860	31.4		1970	203.3	
1870	38.6		1980	226.5	
1880	50.2		1990	248.7	
1890	63.0		2000	281.4	

Source: U.S. Bureau of the Census



Trends, or patterns, are used by planners to predict and prepare for future events.

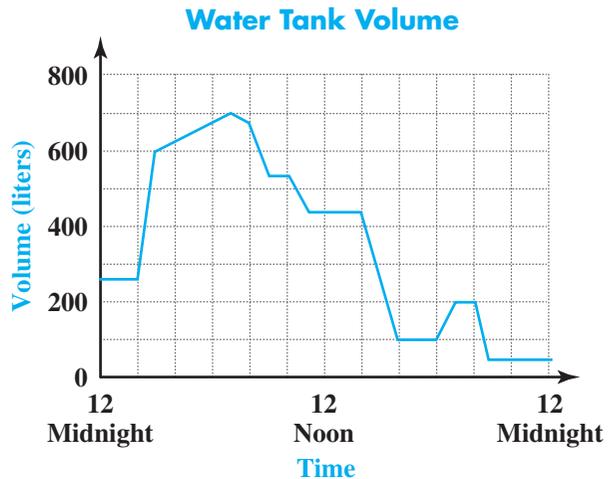
- Copy and complete the table, adding the percentage increase from one time interval to the next.
- Which 10-year interval shows the greatest percentage increase?
- Notice that the increase from 1860 to 1870 is lower than the previous increases. What historical event might have caused this?
- The increase from 1930 to 1940 is very low. What historical event might have caused this?
- Make a graph of the data, with year on the horizontal axis and population on the vertical axis.
- Use the trend of your graph to predict the U.S. population in the year 2050.



In your own words

If you have a graph of a quantity that changes over time, what can you look for to tell how fast or slowly the quantity is changing? How do you know whether the quantity is changing quickly or slowly?

10. Tina Lao lives with her parents, two siblings, and her mother's parents. The Laos have a water tank on their farm property for their household water use. The graph shows how the amount of water in the tank changed over one day.

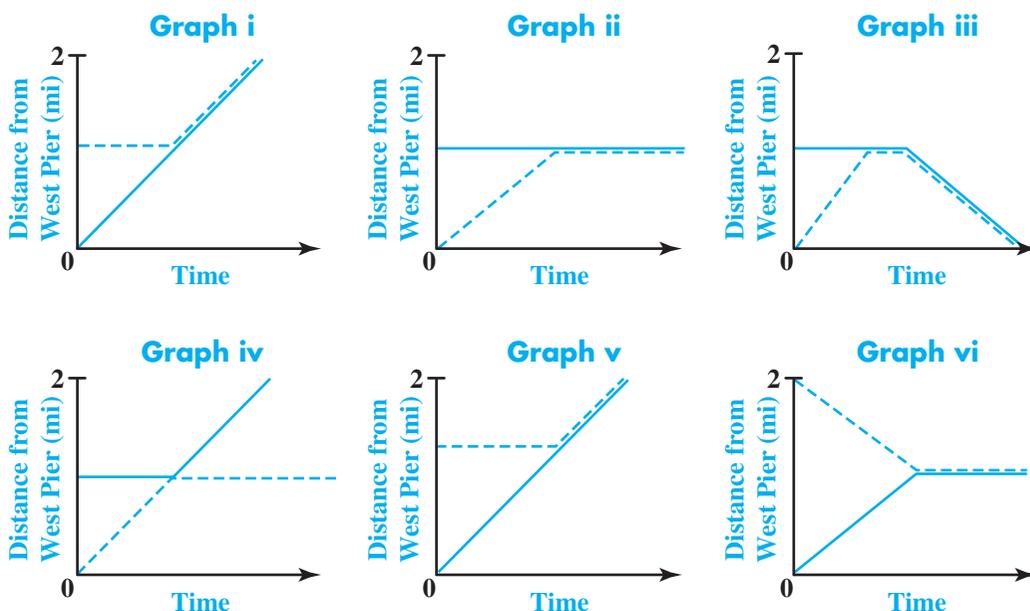


- Write a few paragraphs about what events might have happened during the day to explain parts of the graph. Be creative!
 - What total volume of water (in liters) was used or lost from the tank during the day? Explain.
 - How many liters per hour were removed between 2 P.M. and 4 P.M.?
11. The cylindrical water tank at Ramona's mountain cabin has a diameter of 2 m and a height of 3 m. Suppose the tanker truck fills the tank at a rate of 0.072 m^3 per minute.
- Find the tank's volume. How long will it take to fill it?
 - Carefully draw a graph (not a sketch) showing the *volume* of water in the tank as time passes, until the tank is full. Be sure to show the scales on both axes.
 - Now draw a graph showing the *height* of the water in the tank as time passes, until the tank is full. Show the scales on both axes. Hint: What is the height when you first begin filling? When is the tank full, and what is the height then?
 - Use your graph from Part c to find the height of the water after 1 hour.
 - Use your graph from Part b to find the volume of the water after 1 hour.
 - Challenge** Think of another way to find the volume of the water after 1 hour. How can you use that volume to check your answer to Part d?





- 12.** Two friends are sailing their boats, *Free Time* and *Relaxation*, in a lake. The lake's two piers are at each end of the widest section, a 2-mile stretch. The graphs show the boats' distances from the west pier. The solid line shows *Free Time*'s distance; the dashed line shows *Relaxation*'s.



- Describe what each graph shows. Begin by stating where each boat is at the start.
 - The scales are the same for each graph. Explain what the steepness of a given part of the graph tells you about the boat's speed.
 - How does the slope tell you which direction a boat is sailing?
- 13.** The train schedule shows station arrival times for the three trains on the *Puffing Billy* line between 11:40 A.M. and 1:08 P.M.

Station	Train A	Train B	Train C
Belgrave	leave 11:40 A.M.	12:34 P.M.	1:08 P.M.
Trestle Bridge	11:42 A.M.	12:29 P.M.	1:03 P.M.
Selby	11:50 A.M.	12:26 P.M.	1:00 P.M.
Menzies Creek	arrive 12:00 noon leave 12:17 P.M.	12:14 P.M.	12:48 P.M.
Clematis	12:23 P.M.	12:08 P.M.	12:38 P.M.
Emerald	arrive 12:32 P.M. leave 12:41 P.M.	leave 12:05 P.M. arrive 12:00 noon	12:35 P.M.
Nobelius	12:47 P.M.	11:52 A.M.	12:27 P.M.
Lakeside	12:51 P.M.	leave 11:45 A.M.	leave 12:20 P.M.

Mixed Review

- Draw a graph of the data in the schedule. Put time on the horizontal axis, from 11:30 A.M. to 1:30 P.M. Label the vertical axis with the station names as done in the graph in Problem Set G.
- Write some questions that could be answered using your graph. Be sure to include answers.

Evaluate each expression without using a calculator.

14. $0.51 - 1.2$

15. $3 - 0.006$

16. $0.015 - 0.0015$

Solve each equation.

17. $3(2a - 1) = 3$

18. $2(3 - 3b) = -7.2$

19. $\frac{3a + 4.5}{1.5a} = 2.5$

Find each percentage.

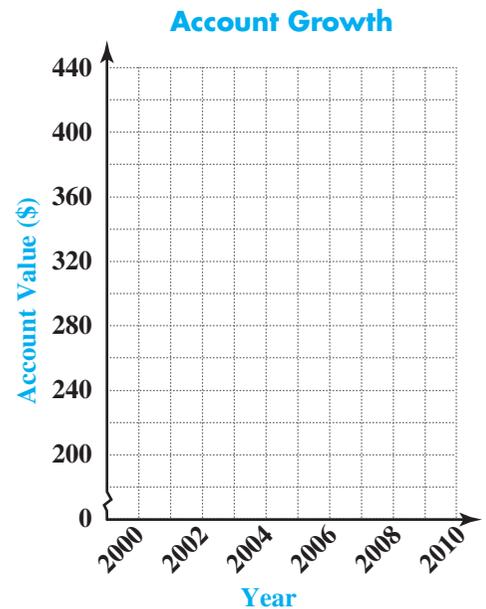
20. 32% of \$100

21. 80% of \$200

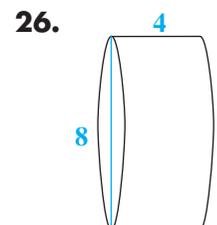
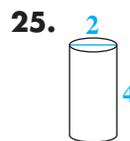
22. 2.5% of \$10,000

23. Suppose Laura invests \$200 on January 1, 2000, at 8% interest per year. Her original investment and the yearly interest are reinvested each year for 10 years.

- How much money is in Laura's account at the end of 2000 (to be reinvested for 2001)?
- Make a graph, on axes like those at right, showing the growth of the account's value for the 10 years the money is invested.



Geometry Find the volume of each cylinder.



9.2

Graphs and Equations

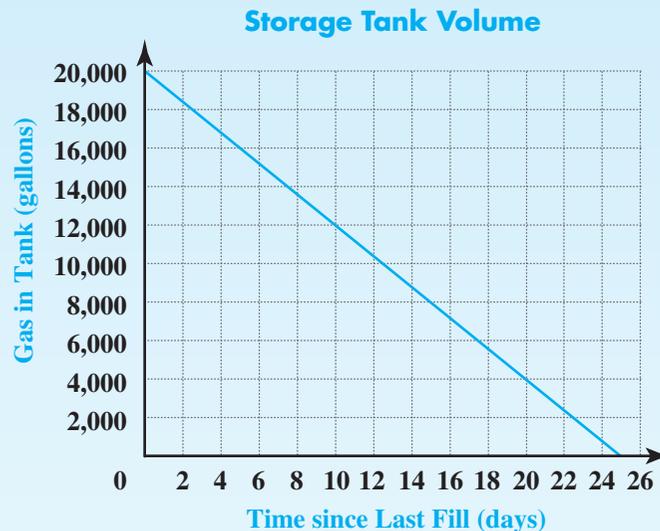
You have investigated many real-world situations that can be modeled by linear relationships. In this lesson, you will encounter three more common types of relationships. As with linear relationships, their graphs have special shapes and their equations have particular forms.



Think & Discuss

A gas station has a 20,000-gallon underground storage tank. The owners know that the amount of gas they sell varies from day to day. By looking at their sales over several weeks, though, they conclude that they sell approximately 800 gallons per day.

The graph below can be used to keep track of about how much gas is left in the storage tank in the days after it has been filled.



How can the graph be used to find the amount of gas remaining in the tank for any number of days since it was last filled?

How long does it take to empty the tank after it has been filled?

How is this shown on the graph?

Write an equation for estimating the amount of gas g in the tank d days after it has been filled.

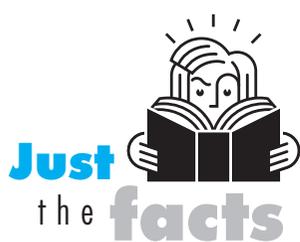
Investigation 1 **Graphs for Squares**

In the Think & Discuss, you inspected a graph of a linear relationship. You know that linear relationships can be expressed symbolically, using an equation in the form $y = ax + b$.

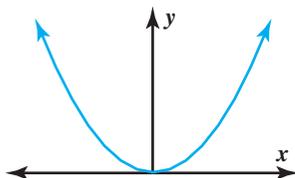
In this investigation, you will explore a different kind of relationship. As with linear relationships, all relationships of this type can be expressed using equations with a similar form, and their graphs have similar shapes.

Problem Set **A**

1. Use your calculator to find the value of x^2 for various values of x . Choose 10 values of x from -3 to 3 . Record your results in a table.
2. Draw a graph of the values in your table. Put x values on the x -axis and x^2 values on the y -axis. Before you start, decide on a reasonable scale.
3. Do your points seem to form a pattern? If so, describe the pattern.
4. Is it sensible to connect the points on your graph? Why or why not? If it does make sense, connect them by drawing a *smooth curve* through them, rather than drawing line segments between them.
5. Write the equation that your graph shows.
6. Extend the curve you drew so you can use it to estimate 3.2^2 . Check your answer with your calculator.
7. Use your graph to estimate the square roots of 7. That is, estimate the values of x for which $x^2 = 7$. Explain what you did to find your answer.



The graph of $y = x^2$ is called a *parabola*.



The paths made by the water in the photograph are also parabolas.



Problem Set B

Simon and Shaunda were playing *What's My Rule?*



1. Write Shaunda's rule in words.
2. Choose two values for n , one positive and one negative, and check that you and your partner agree on outputs for them.
3. Draw a graph for Shaunda's rule, using 10 inputs from -4 to 4 . If it makes sense, connect the points using a smooth curve.
4. How is your graph like the one you drew in Problem Set A? How is it different?
5. When it was his turn, Simon used the rule $output = n^2 + 1$. Discuss with your partner what the graph for this rule might look like. Would it be similar to any graphs you have seen before? If so, which?

Share & Summarize

Describe the shape or pattern of the curve that results when an input variable is squared.

Investigation 2 Graphs for Inverses

VOCABULARY

multiplicative inverse

The next type of relationship you will look at uses the *multiplicative inverse* of a quantity. The **multiplicative inverse** of a number is the result when 1 is divided by the number. For example:

- The inverse of 4 is $\frac{1}{4}$.
- The inverse of -13 is $-\frac{1}{13}$.
- The inverse of x is $\frac{1}{x}$.

MATERIALS

graph paper

Problem Set C

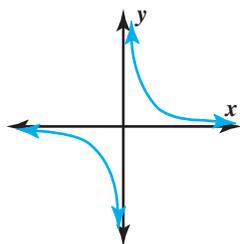
1. Complete the table, showing the values of $\frac{1}{x}$ (that is, $1 \div x$) for various values of x .

x	-4	-2.5	-2	-1	-0.1	0.25	1	1.5	2	3	4
$\frac{1}{x}$											

2. Explain why you cannot use $x = 0$ as an input.
3. Graph the values in your table, with $\frac{1}{x}$ on the y -axis. Make the x -axis from -4 to 4 . Decide on a good scale for the y -axis.
4. Write the equation that your graph shows.
5. Since $\frac{1}{x}$ cannot be evaluated for $x = 0$, the graph of $y = \frac{1}{x}$ cannot cross the y -axis. The parts of the graph on either side of the y -axis can't be connected.
 - a. Draw a smooth curve through the points that fall to the left of the y -axis to show the pattern. It may help to plot more points if you are having trouble deciding on the shape of the graph.
 - b. Now draw a smooth curve through the points that fall to the right of the y -axis. Again, you may need to plot more points.
6. Use your graph to estimate the multiplicative inverse of 2.8 . Check your answer with your calculator.



The graph of $y = \frac{1}{x}$ is called a *hyperbola*.



Problem Set D

Kenneth thought of a new *What's My Rule?* game.



1. Write Kenneth's rule in words.
2. Choose two values for n , one positive and one negative, and check that you and your partner agree on outputs for them.
3. Draw a graph for Kenneth's rule, using 10 inputs from -4 to 4 . If it makes sense, connect the points using a smooth curve.
4. How is your graph like the one you drew in Problem Set C? How is it different?
5. In another game, Liani used the rule $output = \frac{12}{n}$. Discuss with your partner what the graph for her rule might look like. Would it be similar to any graphs you've seen before?

Share & Summarize

1. Why does the graph of $y = \frac{1}{x}$ have two parts?
2. Imagine starting at Point $(1, 1)$ on the graph of $y = \frac{1}{x}$ and moving along the graph so that the x values increase. What happens to the y values?

Investigation 3 Using Graphs to Estimate Solutions

You have learned how to solve equations using backtracking, guess-check-and-improve, and by doing the same thing to both sides. You have also used graphs to estimate solutions to problems.

For example, in Problem Set A, you used your graph of $y = x^2$ to estimate the values of x for which $x^2 = 7$. To make a good estimate from a graph, you need to draw the graph as accurately as possible.

MATERIALS
graph paper

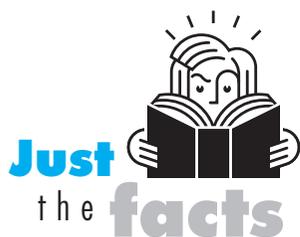
Problem Set E

This little-known formula allows you to estimate the air temperature by measuring the speed of ants crawling around:

$$t = 15s + 3$$

where t is temperature in $^{\circ}\text{C}$ and s is the ants' speed in cm/s.

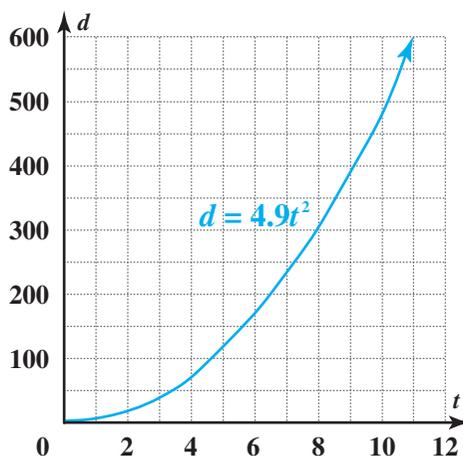
1. Draw a graph to show the relationship between temperature and speed given by the formula. Put s on the horizontal axis with values from 0 to 3.
2. Andrea timed some ants and estimated their speed to be 2.5 cm/s. Use your graph to estimate the temperature.
3. Early in the morning, Andrea estimated the ants' speed to be about 1.2 cm/s, but by late afternoon it was 2.0 cm/s. Use your graph to estimate the change in temperature during that time.
4. Write an equation you could solve to find an ant's speed when the temperature is 25°C . Use your graph to estimate the solution to your equation.
5. Use your graph to estimate how much the ants' speed would change if the temperature increased from 10°C to 20°C .



A temperature of 10°C is equivalent to 50°F .
A temperature of 25°C is equivalent to 77°F .

Problem Set F

When an object is dropped, the formula $d = 4.9t^2$ shows how the distance in meters that the object has fallen is related to the time in seconds since it began to fall. Below is a graph of this relationship.



1. A child dropped a toy from a balcony, 60 meters above the ground. Write an equation you could solve to find how long it will take for the toy to hit the ground.
2. Use the graph to estimate a solution of your equation. Try to give your answer accurate to the nearest tenth of a second.
3. Solve your equation. Round your answer to the nearest tenth of a second. Is the answer close to your estimate?
4. Use the graph to estimate the solution of each equation, accurate to the nearest tenth of a second.

a. $100 = 4.9t^2$

b. $490 = 4.9t^2$

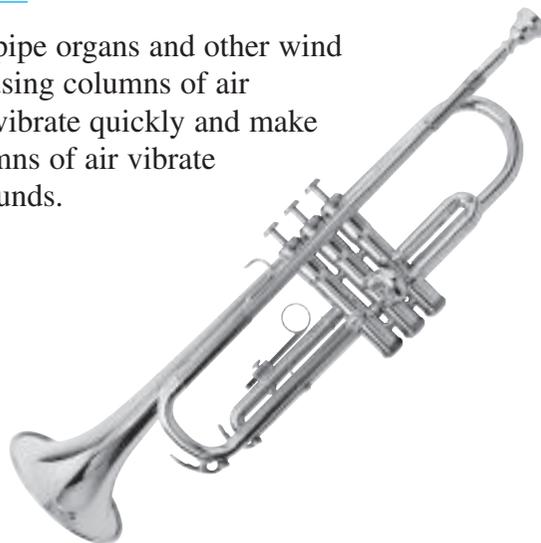
c. $20 = 4.9t^2$

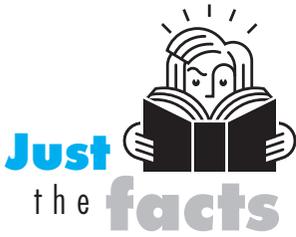


Flutes, clarinets, and trombones are wind instruments: air is blown into them to create sound. The organ is another wind instrument: pressing keys on the keyboard sends air through pipes, producing music.

Problem Set G

Some musical instruments, like pipe organs and other wind instruments, create sound by causing columns of air to vibrate. Short columns of air vibrate quickly and make high-pitched sounds. Long columns of air vibrate slowly and make low-pitched sounds.



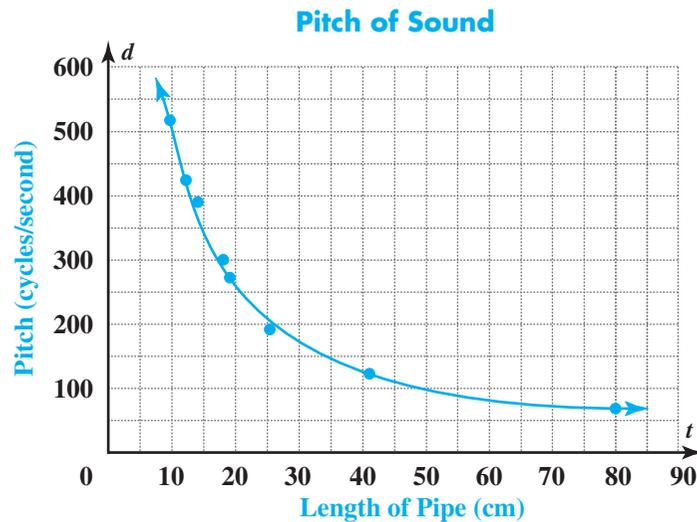


The human voice is also a wind instrument. To produce a singing tone, the diaphragm forces air from the lungs, up through the larynx, and into the mouth. The varying tensions of the vocal chords in the larynx produce different pitches.

Belita was trying to make a musical instrument from a series of pipes of different lengths. She wanted to find a mathematical relationship between the sound and the length of pipe. The *pitch* of a sound tells how high it is. Belita used a device that measures pitch. She blew into pipes of various lengths and recorded the pitches in a table.

Pitch (cycles per second)	64	128	192	261	300	395	438	512
Length of Pipe (centimeters)	80	41	26	19	18	13	12	10

Belita made a graph from her data, adding a smooth curve to fit the points.

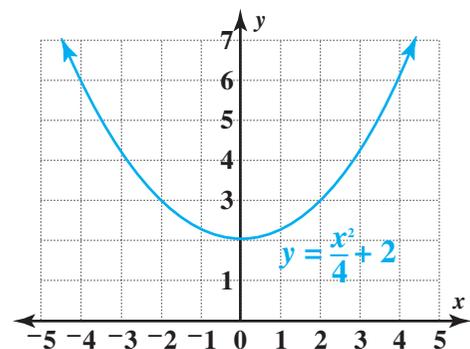


1. Explain how Belita could justify drawing a curve through the points even though she only knew what happened for some pipe lengths.
2. Does the graph look like any of those you have studied in this lesson? If so, which ones?
3. Use the graph to predict the length of pipe Belita needs to produce a pitch of 160 cycles/second.
4. Use the graph to predict the pitch of a pipe 60 cm long.

Share & Summarize

Here is a graph of $y = \frac{x^2}{4} + 2$.

1. Use the graph to estimate the solution of the equation $6 = \frac{x^2}{4} + 2$.
2. Explain how you found your answer.



Investigation 4 Doubling Up



Remember

A quantity grows exponentially if it is repeatedly multiplied by a number greater than 1.

In Chapter 3, you looked at situations that showed *exponential growth*. For example, you may remember the legend of the ruler who gave a subject a grain of rice on the first square on a chessboard. On each square after that, the ruler placed twice as many grains as on the previous square.

In this investigation, you will explore graphs of such relationships.

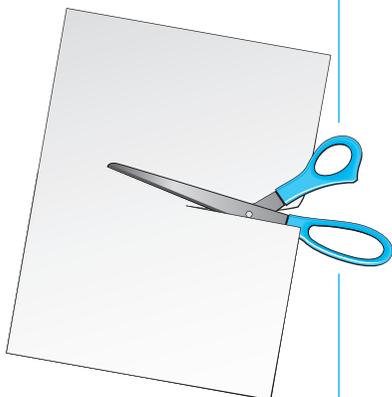
Think & Discuss

Try folding a large sheet of paper in half, then in half again, and then in half again, for a total of eight folds.

Are you able to fold the paper eight times? Why or why not?

MATERIALS

graph paper



Problem Set H

1. Suppose Kyle takes a sheet of paper and cuts it in half. Kyle then cuts both of those pieces in half. How many pieces would he have now?

2. Copy and complete the table.

Stage	0	1	2	3	4	5
Pieces	1	2				

3. Could the relationship between the stage and the number of pieces be linear? Why or why not? What is the pattern in the table?

4. Graph the data. Should you connect the points? Explain. If it makes sense to connect the points, draw a solid curve through them. Otherwise, draw a dashed curve through them.

5. Write a sentence or two describing the shape of your graph.

6. Consider the pattern in the table you created. You can describe the pattern with the equation $p = 2^t$, where t is the stage number and p is the number of pieces. The numeral 2 in the equation shows that doubling is involved. Use the equation to find the number of pieces after 10 rounds of cuts.

7. How many pieces would there be if Kyle could do 20 stages of cuts?

Problem Set

In the late 1700s, before Uranus, Neptune, and Pluto had been discovered, Johann Daniel Titius and Johann Elert Bode made a conjecture that there existed a rule that would predict how far a planet was from the sun.

First they numbered the planets. They started with Earth as Planet 1, moving away from the sun as the planet number increased. Since Venus and Mercury are closer to the sun than Earth is, Venus is Planet 0 and Mercury is Planet -1 .

The distances from the planets to the sun are often expressed in *astronomical units*, or AU for short. One astronomical unit is the average distance from Earth to the sun. (The average is used because the distance varies during the year.)

1. Copy and complete the table, which shows how Titius and Bode predicted the distances from the sun. When they first invented the rule, they found it worked only if they ignored 3 when numbering the planets. Skip this row for now; you will complete it later.

Distance from the Sun

Planet	Planet Number, n	Predicted Distance, d
Mercury	-1	$0.4 + 0.3 \cdot 2^{-1} = 0.55$ AU
Venus	0	$0.4 + 0.3 \cdot 2^0 = 0.7$ AU
Earth	1	$0.4 + 0.3 \cdot 2^1 =$
Mars	2	$0.4 + 0.3 \cdot 2^2 =$
Jupiter	4	$0.4 + 0.3 \cdot 2^4 =$
Saturn	5	$0.4 + 0.3 \cdot 2^5 =$

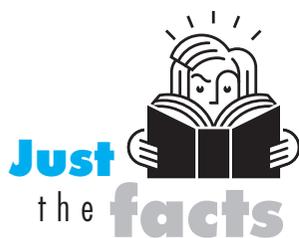
2. What is the rule? Write it first in words, and then in symbols.

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Tohono O'Odham Reservation, Arizona

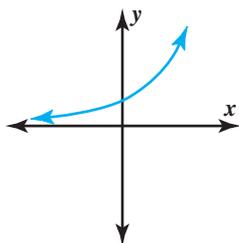


Remember

A dashed line is often used to show the shape of a curve even though the in-between points have no real meaning.



The graph of $y = 2^x$ is called an exponential curve.



3. After Titius and Bode found their rule, astronomers used it to search for a planet with the number 3. This led to the discovery in 1801 of Ceres, an asteroid in the asteroid belt between Mars and Jupiter. Predict the distance the asteroids are from the sun, and complete Row 3 of your table.
4. In 1781, Uranus was discovered using Titius and Bode's rule while searching for planet number 6. Predict its distance from the sun, and add a line to your table for this planet.
5. Plot the points from the table on a grid, and draw a dashed line through them to make a smooth curve.
6. The table gives the actual average distance from the sun to each planet, as calculated by techniques that bounce radio waves off objects in space and measure the time it takes for the signals to return. On the same set of axes, plot points to represent the actual distances. Does the rule seem to give a result close to reality?

Distance from the Sun

Planet	Planet Number, n	Actual Distance, d
Mercury	-1	0.39 AU
Venus	0	0.72 AU
Earth	1	1.0 AU
Mars	2	1.5 AU
Asteroids	3	2.9 AU
Jupiter	4	5.2 AU
Saturn	5	9.5 AU
Uranus	6	19.2 AU

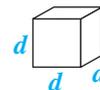
Share & Summarize

1. How does the graph of the doubling pattern differ from a linear pattern?
2. The equation for a doubling pattern has the form $y = 2^x$. The equation for a linear pattern has the form $y = ax + b$. Use these equations to explain the difference in the graphs.

On Your Own Exercises

Practice & Apply

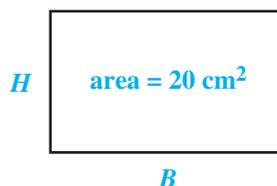
1. **Geometry** The surface area (in square centimeters) of a cube with an edge length of d cm is given by the formula $A = 6d^2$.



- a. Use the formula to complete the table.

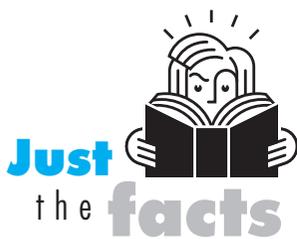
d (cm)	0	0.5	1	1.5	2	2.5	3
A (cm ²)							

- b. Plot the points in the table. If it is reasonable to do so, connect them with a smooth curve.
- c. Compare your graph to the graph of $y = x^2$, which you created in Problem Set A.
2. For a game of *What's My Rule?* Takiyah decided to use the rule $output = n^2 - 3$.
- a. What shape will the graph of Takiyah's rule have?
- b. Without plotting any points, make a rough sketch of the graph of Takiyah's rule.
3. **Geometry** Melisenda and Jade were drawing rectangles, each with an area of 20 square centimeters.



- a. Write a formula for finding the base B given the height H .
- b. What values make sense for H ? Must H be a whole number? Can it be negative?
- c. Use your calculator to find at least 10 possible values for B and H , including decimals or fractions if they make sense.
- d. Draw a graph of the data from your table.
- e. Would it make sense to connect the points on your graph? Explain. If it makes sense, do so.
4. Malik's rule for *What's My Rule?* was $output = \frac{20}{n}$. Without plotting points, make a rough sketch of the graph of Malik's rule.

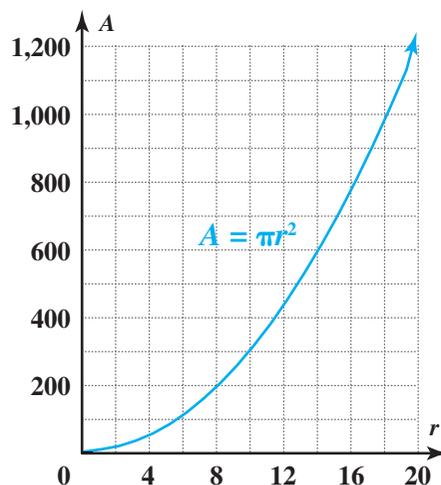




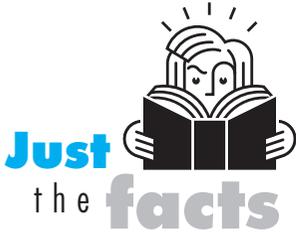
In 1999, the *euro* was introduced as a new currency to be used by all countries in the European Union.



5. The currency in the United Kingdom is the pound (symbol £). On a particular day in June 2003, the rule for converting pounds to U.S. dollars was $D = 1.65P$, where D was the price in U.S. dollars and P was the price in pounds.
 - a. Draw a graph of this relationship. Put P on the horizontal axis, and show amounts in pounds up to £150.
 - b. On that day, a British collector on the Internet advertised a set of miniature cars for £125, including postage. Write an equation you could solve to find how much this was in U.S. dollars, and solve your equation. Use your graph to check your solution.
 - c. On that same day in June, Megan exchanged \$120 she had saved toward a holiday in Wales. Write and solve an equation to find how many pounds she received. Use your graph to check your solution.
 - d. What are the advantages of using either the graph or the rule to solve the equations you wrote?
6. **Geometry** The area A of a circle of radius r can be found using the formula $A = \pi r^2$. This graph shows the relationship.



- a. A pastry recipe claims that a certain quantity of dough will roll out to an area of 1,000 square inches. A cook plans to make a large circular pie crust. Write and solve an equation to find what radius he can expect the pie crust to have. Use the graph to check your solution.
- b. A pizza shop sells a 10-inch pizza for \$6.50 and a 20-inch pizza for \$15.00. Use the graph to help you decide which pizza costs more per square inch of pizza. Explain.



Many different mathematical models and techniques are used to estimate the world population. We will never know for certain how many people were actually on the planet at any particular time.

7. Scuba divers usually descend to a desired depth and spend some time exploring there. Although the amount of oxygen needed varies from person to person, the following rule gives an estimate of how long an average diver can stay at a particular depth. T is the maximum time under water in minutes, V is the volume of air available in cubic meters, and D is the depth in meters.

$$T = \frac{120V}{D}$$

- Michael has 1 m^3 of air in his tank. Write an equation to show the relationship between the time T he can stay under water and the depth D to which he can dive.
- Find 10 possible pairs of values for T and D for Michael's equation. Draw a graph of the relationship.
- Use your graph to find how long Michael can stay at a 25-m depth. Check your answer using the equation you wrote in Part a.
- Use your graph to find how deep Michael can dive if he wishes to stay under water for 15 minutes at the maximum possible depth. Check your answer by using the formula.

8. **Social Studies** The table shows the United Nations estimates for the world population from 1750 to 2000.

World Population

Year	Population (millions)
1750	790
1800	980
1850	1,260
1900	1,650
1950	2,520
1970	3,700
1990	5,270
2000	6,100

- Plot these data on a graph. Put the year on the horizontal axis, from 1750 to 2100, and make the scale on the vertical axis go up to 10 billion people. Draw as smooth a curve as you can through your points, extending the curve beyond 2000.
- Use your graph to estimate the world population in the year you were born and in the year you will turn 18. What is the likely increase in population over that period?
- Use your graph to estimate the 1980 world population.
- The actual 1980 world population was about 4,440 million, or 4.44 billion. How close was your estimate?

Connect & Extend

9. Businesses often set prices for their products. Businesses would like to sell many products at a high price—but if the price is too high, people won't buy the products.

Kate and her friends earn money by selling greeting cards they create on a computer. To help decide how much to charge, they surveyed 100 people about the price they would pay for a card. They organized the information in a table.

Price (\$)	0	1	2	3	4	5
Number of People Surveyed Who Would Buy One Card	100	80	60	40	20	0
Income (\$)						

- Copy the table, and complete it with the income the friends could make for each price. For example, if they set the price at \$1.00, they could sell 80 cards to these 100 people for a total of \$80. Explain how you found your answers.
 - Make a graph showing the income for the various prices. Connect the points in your graph with a smooth, dashed curve. What shape does the curve have?
 - Look closely at the curve you drew. What price do you think the friends should charge to earn the most income?
 - Write an expression that gives the number of cards sold for any price p .
 - If the friends sell n cards for p dollars each, what will their income be?
 - Replace the n in your expression from Part e with the expression you wrote for Part d. Use the distributive property to rewrite your new expression.
 - Consider your rewritten expression in Part f and the graph you made in Part b. How do the expression and the graph compare to the equations and graphs from Investigation 1?
10. **Geometry** Sean and Fiona are planning a vegetable garden. They figure they need 36 square yards to grow all they would like. They want the garden to have a rectangular shape, but they can't decide on its length and width.
- Find at least 10 sets of lengths and widths the friends could use to get a total area of 36 yd^2 . Use your data to make a graph, with length on the horizontal axis. Connect the points to form a smooth curve, if it makes sense to do so.

Sean and Fiona decided to enclose their garden with a fence to protect it from animals. To keep material costs down, they want the rectangle to have the least possible perimeter.

- b.** Calculate the perimeter of each rectangle you found in Part a. Label each plotted point on your graph with the corresponding perimeter. Do you see a pattern in the labels?
 - c.** What point on the graph do you think would have the least perimeter? What size garden should Sean and Fiona create?
- 11.** A video games arcade has a special deal on school holidays. Instead of paying \$1 for each game, you pay \$2.40 admission and 40¢ per game.
- a.** Copy and complete the table.

Number of Games	1	2	4	7	10
Regular Price (\$)	1.00				
Holiday Price (\$)	2.80				

- b.** Plot the data in your table on one set of axes. Make two graphs that show the costs of playing different numbers of games at the regular price and at the holiday price. Connect the points using solid or dashed lines, whichever make more sense. Explain why the type of lines you chose makes sense.
- c.** Find an equation for the cost of playing any number of games at the holiday price. Use your equation to determine how much it would cost to play three games.
- d.** Suppose you have \$6 to play video games on a holiday. Write an equation you could use to determine how many games you could play. Use your graph to estimate the solution of the equation.
- e.** For which numbers of games are the holiday prices lower than the regular prices? For which numbers of games are they higher?

- 12. Physical Science** David and Angela were experimenting with gravity and falling objects. They dropped stones from four heights and carefully timed how long it took them to hit the ground.

These are their results.

Height (ft)	0	5.2	11.8	14.7	20.1	28.7
Time (s)	0	0.57	0.86	0.96	1.12	1.34

- a.** Plot the results on a graph with time on the horizontal axis and height on the vertical axis. Connect the points using a smooth curve.

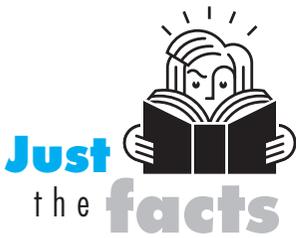
Angela thought the curve looked like the graph of $y = x^2$. Using H for height and t for time, she suggested that an equation in the form $H = Kt^2$ might fit the data.

- b.** Choose a time other than 0 from the table. Substitute the time and the corresponding height into the equation $H = Kt^2$, and solve the new equation.
- c.** Repeat Part b for the other times except 0. What might be a good value for K ?
- d.** Rewrite Angela's equation, using your value from Part c for K .
- e.** Angela and David realized they could use the graph to test their throwing arms. One of them threw a baseball as high as possible. The other watched carefully until the ball reached its highest point, and then timed it until it hit the ground. By looking at the graph, they could figure out how high they had thrown the ball.



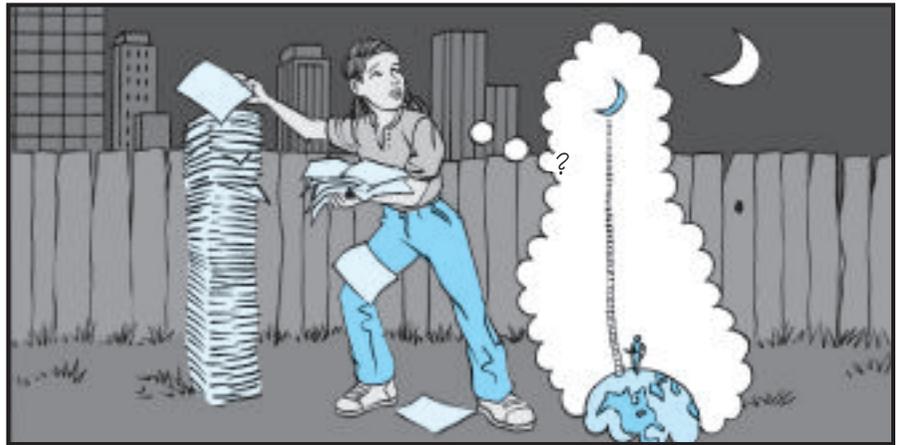
These are the times the baseball took to fall. Use the graph to estimate how high the ball rose each time. Check your answers using your equation from Part d.

Time (s)	1.05	1.20	1.43	1.31
Height (ft)				



The moon's orbit about Earth is elliptical, not circular, so its distance from Earth changes. The nearest it comes to Earth (this point in its orbit is called the *perigee*) is about 363,000 km; the farthest (the *apogee*) is about 405,000 km.

- 13.** Maya took a sheet of paper about 0.1 millimeter thick and cut it in half. She stacked the pieces, one on top of the other.
- How thick is Maya's two-sheet stack?
 - Maya cut each of the pieces in half and stacked all the new pieces. How thick is the new stack?
 - Suppose Maya continues to cut all the pieces, stack them, and then measure the stack's height. Write an equation that gives the thickness t after any number of cut-and-stacks c .



- How thick will the pile be after 10 cut-and-stacks? After 20? Answer first in millimeters, and then convert to meters.
 - How many cut-and-stacks will Maya need to make a thickness of at least 1 kilometer? How thick will the pile be?
 - The moon is about 400,000 km from Earth. Find the number of cut-and-stacks needed for Maya's pile of paper to reach the moon.
- 14.** Cut a long strip about 1 centimeter wide from a sheet of paper with no creases. Fold it in half, end to end, once. When you open it, there will be one crease.
- Continue to fold the strip in half, end to end. At each stage, open it and count how many creases it has. Complete the table.

Times Folded, f	0	1	2	3	4	5
Creases, c	0	1				

- Graph this relationship.
- Write an equation to describe the relationship between the number of creases and the number of folds. Hint: It may help you see the pattern if you add 1 to each of the c values.

In your own words

Explain how a graph can help you solve an equation. Make up a problem of your own to use as an example.

- 15.** Julian and Gina conducted a game with their class of 32 students. The students divided into 16 pairs. One student in each pair tossed the coin, while the other called “heads” or “tails.” The 16 winners of the tosses paired off and played again.

This knockout competition continued until only one person was left. That person was declared the winner of the game.

- How many rounds were played before the game winner could be determined?
- If the game is played a second time, do you think the winner from the first game is more likely, just as likely, or less likely to win the second game than anyone else in the class? Explain.
- Suppose the students tried the game with the entire school population of 512 students. How many rounds would it take to find the winner?
- Supposed they tried it in a city of 1,048,576 citizens. How many rounds would it take to find the winner?
- About how many rounds would it take to find the winner if they could get the whole world to play? Assume the world population is about 6 billion.

Mixed Review

Evaluate each expression.

16. $\frac{11}{12} - \frac{7}{12} + \frac{2}{3}$

17. $\frac{2}{7} + \frac{1}{8} + \frac{1}{2}$

18. $\frac{2}{9} - \frac{1}{7} + \frac{1}{3}$

19. $\frac{1}{6} + \frac{1}{2} - \frac{1}{12}$

Write each number in scientific notation.

20. 237

21. 637,800

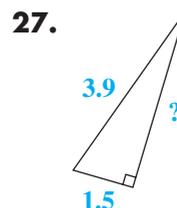
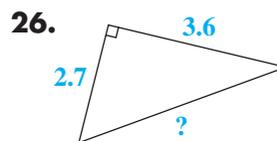
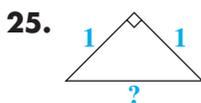
22. 0.057

Evaluate each expression. Write your answers in standard notation.

23. $(4.5 \times 10^{-3}) - (2.5 \times 10^{-5})$

24. $(1.7 \times 10^{-4}) + (9.2 \times 10^{-5})$

Geometry Find each missing side length.



Nutrition Information

Serving Size:

4 cookies (28 g)

Calories: 120

Amount per Serving

Total Fat 4 g

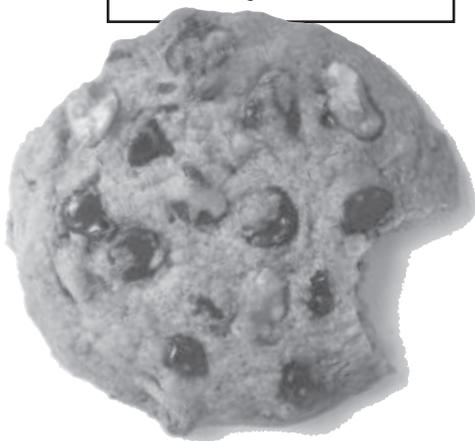
Saturated Fat 2 g

Total Carbohydrate 19 g

Dietary Fiber 0 g

Sugars 7 g

Protein 1 g



28. Nutrition information for Damon's favorite cookies are shown on the label.

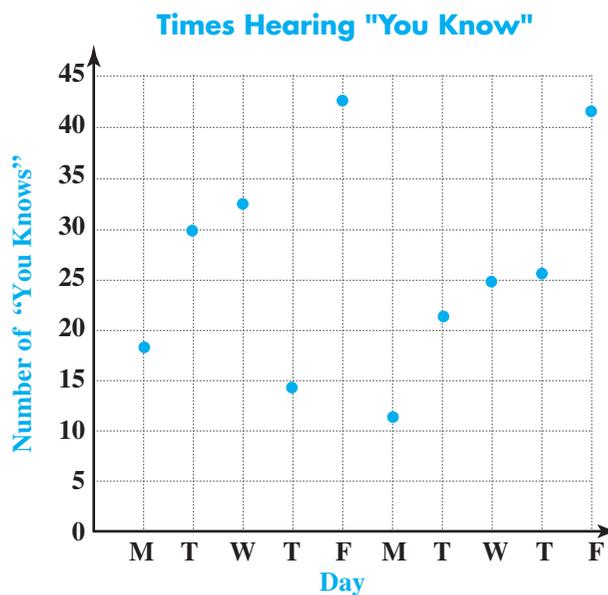
- Damon usually eats eight cookies when he comes home from school. How many servings is this, according to the package?
- One gram of fat contains 9 calories. How many calories in one serving of these cookies are from fat?
- What percentage of the calories in one serving of these cookies are from fat? Show how you found your answer.
- One gram of protein contains 4 calories. What percentage of the calories in one serving of these cookies are from protein?
- If Damon decides to consume only 1.5 grams of saturated fat in his afternoon cookie snack, how many cookies can he have?

29. Measurement How many days is 1,000,000 seconds?

30. How many years is 1,000,000 hours?

31. How many decades is 3,500 years?

32. Every day for two weeks, Amato kept track of how many times he heard someone say "you know" during a class discussion. He made a graph of his data.



- How many "you knows" did Amato hear in all?
- On what day of the week did Amato hear the most "you knows"? What percentage of all the "you knows" were heard on this day?

9.3

Repeating Relationships

Think about how many things repeat in your life. Perhaps you have a daily routine. You attend school for five days and then don't for two days. Seasons come and go. In this lesson, you will explore some relationships that repeat.

Explore

Find the missing values in the table for the number the minute hand on a circular clock face would point to for each given time.



Time	Minute-Hand Position
12:00	12
12:15	3
12:30	
12:45	
1:00	
1:15	
1:30	
1:45	

Time	Minute-Hand Position
2:00	
2:15	
2:30	
2:45	
3:00	
3:15	
3:30	
3:45	

You probably noticed that the table shows a repeating pattern. What is the pattern? Explain why it repeats every hour.

Plot the data from the table. Put time on the horizontal axis and minute-hand position on the vertical axis.

Think of some other repeating patterns you could see in a clock, and graphs you could produce from those patterns. Write at least two sets of instructions for producing a clock pattern that repeats, and state how often it repeats.

Investigation 1 Repeating Patterns

In this investigation, you will look at simple repeating patterns to learn something about their graphs.

MATERIALS
graph paper

Problem Set A

Damian started his summer job early, beginning work after school in May. He gets a paycheck every four weeks, which he deposits in his bank account. Each week he takes out \$30 for spending money.

Damian wants to figure out how much money he will have saved by the beginning of his family vacation at the end of July. The table shows his predictions of transactions in his bank account for May through July.

Account Balance

Date	Deposit	Withdrawal	Balance
May 4	\$400	\$30	\$570
May 11		\$30	\$540
May 18		\$30	\$510
May 25		\$30	\$480
Jun 1	\$400	\$30	\$850
Jun 8		\$30	\$820
Jun 15		\$30	\$790
Jun 22		\$30	\$760
Jun 29	\$400	\$30	\$1,130
Jul 6		\$30	\$1,100
Jul 13		\$30	\$1,070
Jul 20		\$30	\$1,040

1. What must the balance have been before May 4?
2. Plot the data in the “Balance” column. Put the date on the horizontal axis and the balance on the vertical axis.

3. Think about whether you should connect the points on your graph. Damian's withdrawals and deposits occurred all at once, not in fractional amounts. That means the graph would have to show sudden changes, not gradual changes.
 - a. How much money would be in the account on May 5, 6, 7, 8, 9, and 10?
 - b. Add points for those days to your graph.
 - c. Does it make sense to connect those points? Explain. If it makes sense, do so.
 - d. Should you connect the points for May 10 and May 11? Explain. If it makes sense, do so.
4. Revise your graph until you are convinced it shows the relationship for *all* times from May 4 to July 26. Discuss it with your partner.
5. What is Damian's balance on July 15? If the pattern continues, what do you expect it to be on August 3? On August 9?

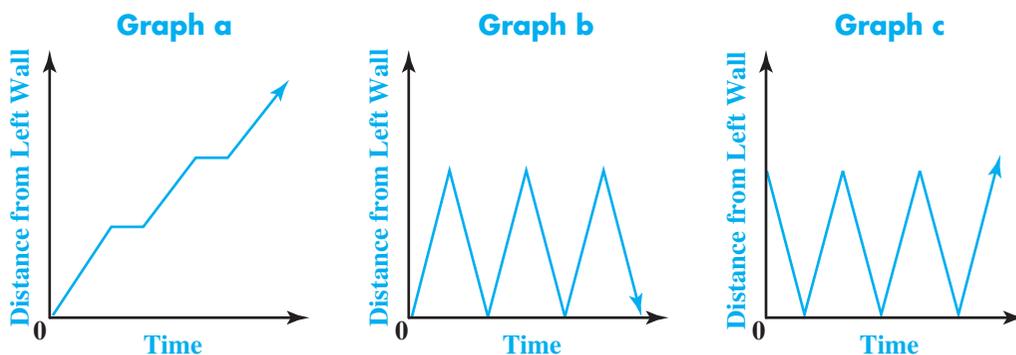
MATERIALS
graph paper

Problem Set B

When Darnell is nervous, he often paces slowly and steadily across a room. Suppose Darnell started from the left wall of his room and walked across to the right wall. When he reached the right wall, he turned immediately around and paced back to the left wall, then walked again from left to right, and so on.



1. Which graph best shows Darnell's distance from the left wall at any time? Why?



2. The other graphs also describe a person walking in a room. Describe the situation shown by each of the other graphs.

A child is playing on a slide. He climbs up, sits down, and then starts to slide, gaining speed as he descends. At the bottom, he gets up quickly, runs around to the ladder, and climbs up again. He does this over and over.

3. Sketch a graph to show how the height of the child's feet above the ground is related to the time.
4. Would your graph look different if, instead of using the height of the child's feet, you used the height of his head? Sketch this graph on the same set of axes as the graph for Problem 3.

Share & Summarize

1. Give an example of a repeating number pattern involving money.
2. What would the graph of your pattern look like?



Investigation 2 Repeating Patterns in Life

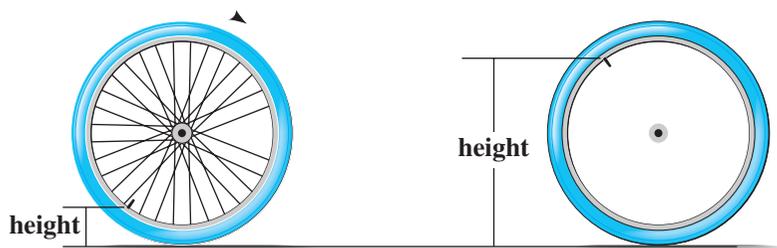
In each repeating relationship you have graphed, the graph showed the same basic shape repeated over and over at fixed intervals of time. The following problems show how one particular kind of repeating pattern can appear in two different situations.

MATERIALS
graph paper

Problem Set C

As a bicycle wheel turns, its tire valve is at different heights from the ground. Imagine the following experiment:

Chalk marks are placed on a mountain bike tire, one mark at every fourth spoke. The bicycle rolls along slowly. Whenever a chalk mark touches the ground, the height of the valve is measured in inches. When the first measurement is taken (Measurement 0), the valve is 8 inches above the ground. The table lists the valve heights from Measurement 0 to Measurement 16.

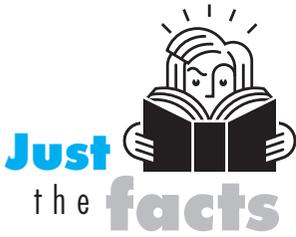


Measurement	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Valve Height (in.)	8	3.5	2.5	8	18	22.5	21.5	15.5	8	3.5	2.5	8	18	22.5	21.5	15.5	8



Most mountain bike wheels have 32 spokes. Wheels are also made with 28 spokes (for someone who wants a lighter wheel) and 36 spokes (for someone who wants a more durable wheel).

1. Draw a graph of the data. Connect the points with a smooth curve.
2. Describe the shape of your graph. Use the tire situation to explain why the graph has this shape.



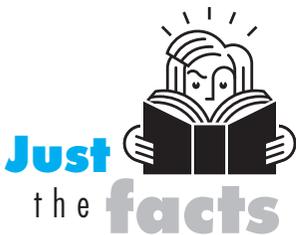
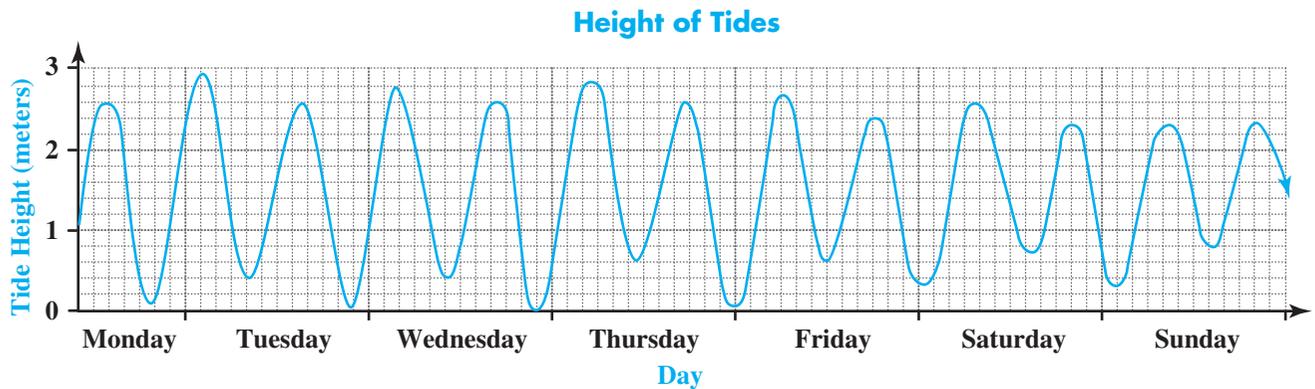
The tides are caused by the pull of the moon's gravity on the ocean water. The change in the tidal pattern is related to the moon's position.

Patterns are very useful for making predictions. For example, the repeating pattern of the tides is so regular that tide times can be predicted for many years ahead. People who live near the sea often use tide tables.

Problem Set D

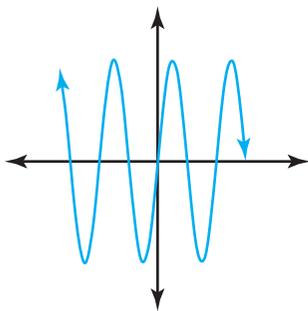
The graph shows the height of tides over a week in a particular seaport. When the height of the water reaches a high point and just starts to recede, the ocean is said to be at *high tide*. When the height reaches a low point, the ocean is at *low tide*. The same basic shape is repeated, but the pattern is not exactly the same each day.

The graph starts on Monday at 10 A.M. Every vertical grid line represents 2 hours.



The wave-like curve below has a special name: a *sine curve*. Sine curves are studied in a branch of mathematics called *trigonometry*.

1. Look at how the tide height changed on Tuesday.
 - a. When were the high tides? Were they the same height? Explain.
 - b. When were the low tides? Were they the same height? Explain.
2. How often does the pattern repeat?
3. For what situations might people need to know the tide patterns?



Share & Summarize

1. Give an example of a repeating pattern you might encounter.
2. How could you make a graph from your pattern? What would the graph of your pattern look like?

Investigation 3 Repeating Patterns in the Weather

Because certain weather patterns repeat year after year, it is possible to describe the typical weather for a location at a given time of year.

MATERIALS
graph paper

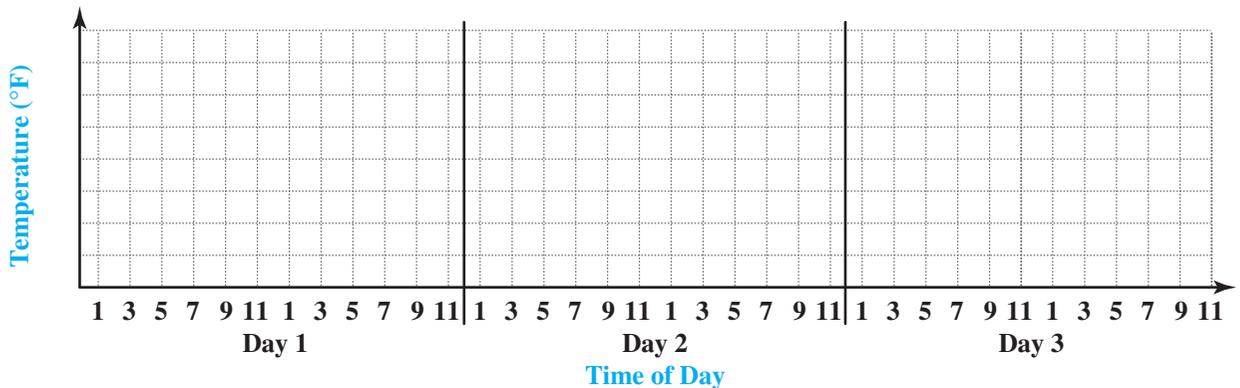
Problem Set E

The table lists the temperatures every 2 hours over a three-day period in a recent month for three cities: Norfolk, Virginia; San Francisco, California; and St. Louis, Missouri.

Source: National Climatic Data Center, *OnLine Climate Data*, www.ncdc.noaa.gov.

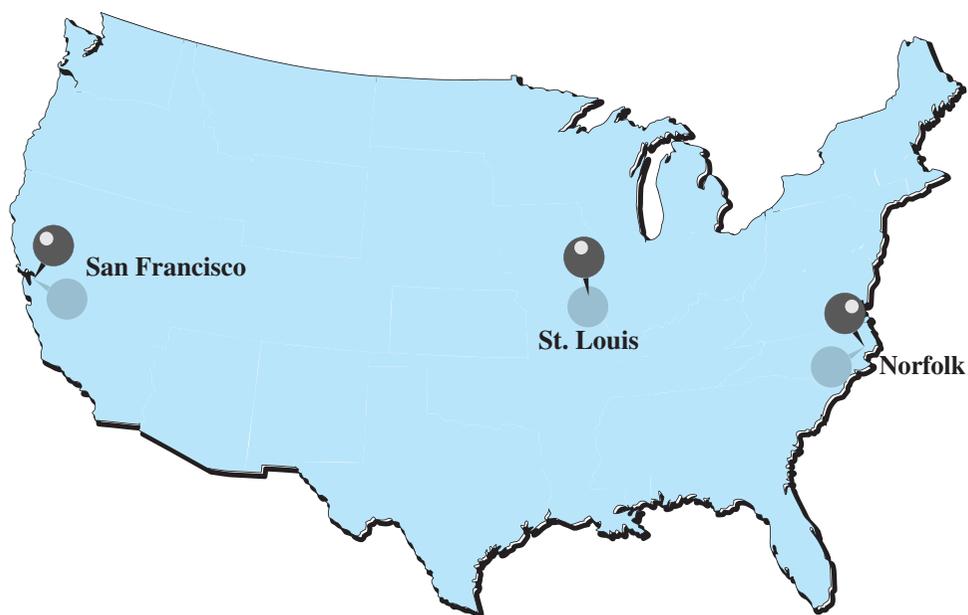
Time	Temperature (°F)								
	Norfolk, VA			San Francisco, CA			St. Louis, MO		
	Day 1	Day 2	Day 3	Day 1	Day 2	Day 3	Day 1	Day 2	Day 3
1 A.M.	37.9	55.0	41.0	45.0	46.9	46.9	63.0	30.0	23.0
3 A.M.	35.1	55.0	41.0	46.0	46.9	45.0	62.1	30.0	21.0
5 A.M.	34.0	55.9	39.0	41.0	46.0	44.1	63.0	28.9	21.0
7 A.M.	35.1	61.0	35.1	41.0	42.1	44.1	66.0	34.0	24.1
9 A.M.	46.0	64.9	37.9	45.0	44.1	48.9	71.1	41.0	28.9
11 A.M.	57.9	66.9	45.0	46.9	46.9	48.9	69.1	43.0	35.1
1 P.M.	60.1	66.0	46.9	50.0	50.0	50.0	60.1	39.9	36.0
3 P.M.	63.0	72.0	45.0	52.0	51.1	55.0	44.1	36.0	37.9
5 P.M.	61.0	70.0	43.0	52.0	51.1	48.9	37.0	33.1	34.0
7 P.M.	55.0	68.0	39.0	51.1	50.0	46.9	34.0	32.0	30.0
9 P.M.	55.9	51.1	34.0	50.0	48.9	45.0	33.1	31.1	28.9
11 P.M.	54.0	45.0	30.9	48.0	48.0	46.9	30.9	30.0	31.1

- Working with your group, graph the data for each city. Put the times for the entire three days on the horizontal axis, and the temperature on the vertical axis. Use the same scales for each of the three graphs so you will be able to compare the graphs later. Label the temperature axis in a way that will account for the range of temperatures in the data. Connect your plotted points with line segments.





2. Explain how your graphs show the following things:
 - a. the daily maximum temperature
 - b. the daily minimum temperature
 - c. when temperatures are changing most rapidly
3. What do the patterns indicate might happen the next day (Day 4) in San Francisco?
4. How are the three graphs similar? How do they differ?
5. Why do you think the similarities you noted occur?
6. Make a rough sketch of the likely temperature patterns for where you live. Your graph should show an estimate of the maximum daily temperatures over an entire year. Discuss your graph with a partner and explain why it is shaped the way it is.



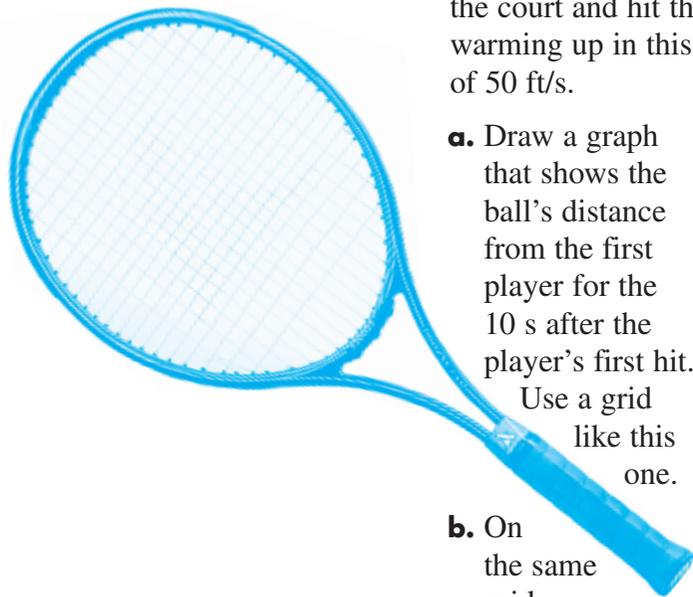
Share & Summarize

1. Describe some repeating patterns involving weather.
2. What advantage does graphing such data have over just using a table?
3. What do the slopes of the line segments in the graphs you created for Problem Set E show?

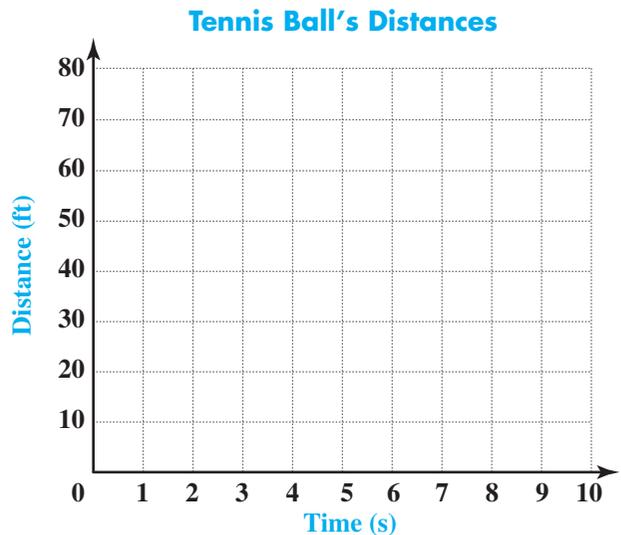
On Your Own Exercises

Practice & Apply

- Every Tuesday Pablo deposits his allowance of \$10 into his bank account. Every Thursday he deposits the \$15 he earns from his part-time job. Then, every Friday, he takes out \$12 for weekend spending.
 - Draw a graph that shows the amount in Pablo's account over the first three weeks (21 days) after opening the account and making his first Tuesday deposit (on Day 1).
 - How often does the graph repeat? How are the repeated sections different from each other?
 - In which week will Pablo's account first reach \$100? Explain.
- Sports** Tennis is played on a court 78 feet long, with a net in the center. To warm up before a game, players will stand at either end of the court and hit the ball back and forth. Suppose two players are warming up in this way. Assume the ball travels at a constant speed of 50 ft/s.

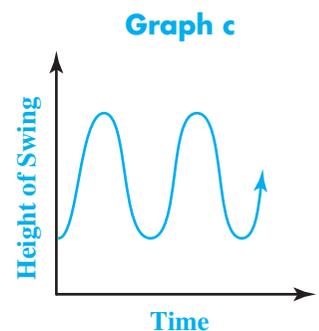
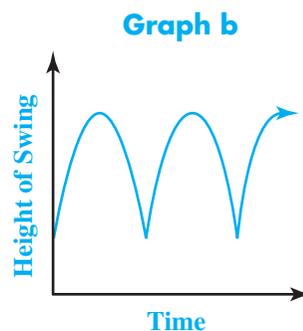
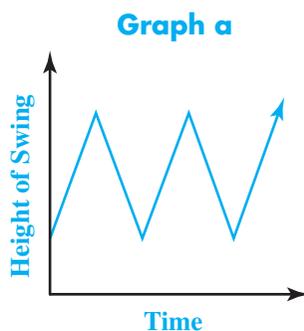


- Draw a graph that shows the ball's distance from the first player for the 10 s after the player's first hit. Use a grid like this one.



- On the same grid, use another color to draw a graph that shows the ball's distance from the *net* over the same period of time.

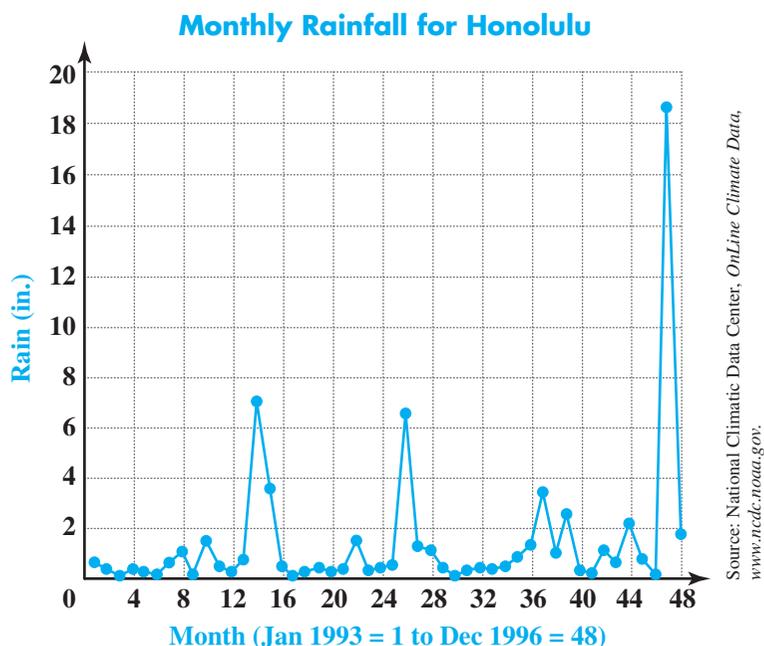
- Tala and her friend Jing were playing on the swings. Tala noticed that Jing went up, came down, and then rose again in a repeating pattern. Which of these three graphs do you think best represents Jing's height over time? Why?



4. When the carnival was in town, Maria watched her sister Rosi ride the carousel. As the carousel turned, Rosi moved away from Maria, and then back toward her again. The table shows how the distance between the girls changed as the carousel revolved. Data for the first 24 seconds of the ride are shown.

Time (s)	0	4	8	12	16	20	24	28	32	36	40	44	48
Distance (ft)	13	7	13	20.2	23	20.2	13						

- Plot the data, with times up to 50 seconds on the horizontal axis.
 - Describe the pattern in your graph. What do you think will happen over the next 24 s?
 - Complete the table, assuming the movement continues in the same pattern.
 - Add the new points to your graph. If it makes sense to do so, draw a smooth curve connecting the points in your graph.
5. **Earth Science** Rain tends to fall at certain times of the year in many areas, and even at certain times of the day in some. The graph shows the monthly rainfall for Honolulu, Hawaii, from 1993 to 1996. January 1993 is Month 1, and December 1996 is Month 48.



- Describe the pattern in the graph.
- Are there any months that don't seem to follow the pattern well? Explain.
- From the pattern in the graph, how often should the people in Honolulu expect a lot of rain? About when would that occur?



- 6. Earth Science** Although the temperature in Hawaii doesn't vary as much as it does in other places, the seasons still have an effect. The table gives Honolulu's average maximum temperature for every other month from 1993 to 1997.

Average Maximum Temperature (°F)

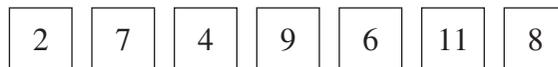
Month	1998	1999	2000	2001	2002
January	81.6	79.6	78.6	82.1	80.2
March	82.3	80.7	82.0	81.6	80.4
May	82.7	83.2	85.1	84.9	83.8
July	85.8	85.8	87.4	87.9	87.1
September	87.8	87.2	87.1	88.8	87.9
November	83.5	83.0	83.1	83.1	84.0

Source: Western Regional Climate Center

- Plot these data, with time on the horizontal axis and temperatures from 75°F to 95°F on the vertical axis. Connect the points with line segments.
- Describe the pattern in your graph.
- At what time of year does Honolulu experience its highest temperatures? Its lowest temperatures?

Connect & Extend

- 7.** Tony solved three pattern puzzles. Each puzzle had seven numbered cards. To solve each puzzle, Tony had to arrange the cards to show a repeating pattern that involved addition and subtraction.
- The first puzzle had seven cards, numbered 2, 4, 6, 7, 8, 9, and 11. Tony's solution is shown below. What is the repeating pattern in this solution?



- The cards in the second pattern puzzle were numbered 7, 8, 9, 10, 12, 13, and 14. Tony found a solution that started with 10 and ended with 7. Find Tony's solution, and describe the repeating pattern.



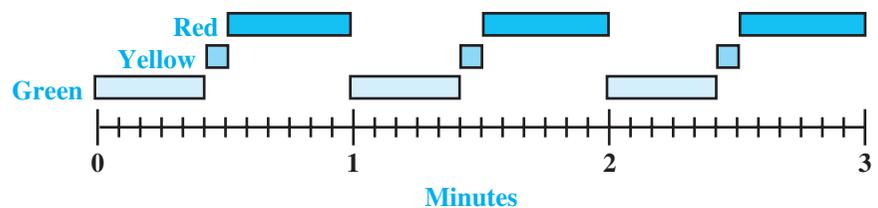
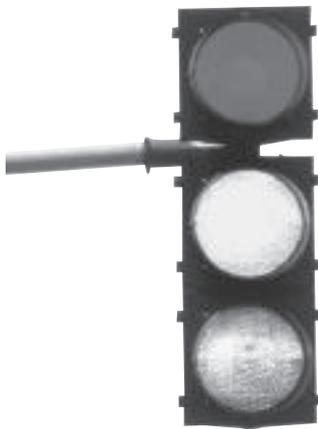
- The third puzzle's cards were numbered 0, 1, 2, 3, 4, 5, and 7. Tony's solution started with 1 and ended with 7. Find his solution, and describe the pattern.



- d. Each of the three patterns can be continued to the right and to the left. For example, in the first puzzle, the first five extra numbers to the right would be 13, 10, 15, 12, and 17. The first five extra numbers to the left, from left to right, would be 1, -2 , 3, 0, and 5. So the fifth extra number to the right is 17 and the fifth extra number to the left is 1.

For each of the other two puzzles, find the fifth extra number to the right and the fifth extra number to the left.

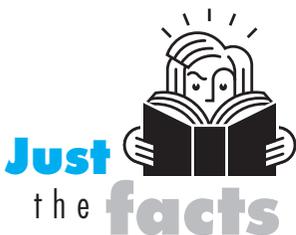
8. Julie and Trevor were timing the traffic signal of one of the lights at the intersection by their school. They drew a timeline of their data.



- a. How many seconds is one complete cycle?
- b. Julie and Trevor noticed that when the light in one direction turned red, there was a delay of about 2 s before the light in the other direction turned green. Why do you think this delay is built into the sequence?
- c. Given a delay of 2 s, how would the timeline for the light in the other direction look? That is, when would it be red, green, and yellow, compared to the timeline for the first light? Create a timeline to show this light sequence. Assume the yellow lights stay on for the same amount of time in each direction.

9. **Astronomy** Halley's Comet is a regular visitor to our skies. It completes its elongated orbit about once every 76 years. Its last visit to Earth was in 1986.

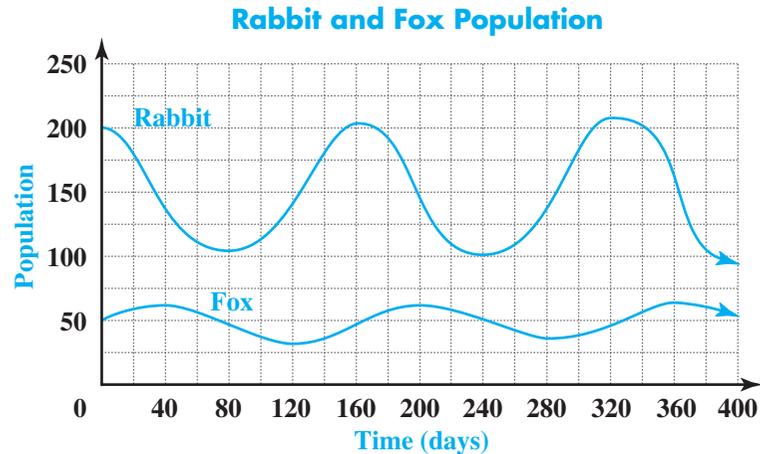
- a. When will the next visit be? How old do you expect to be then?
- b. Find the years of the three appearances of Halley's comet prior to 1986.
- c. Scientists can use patterns to do "detective work" and explain past events. In about 1304, the Italian artist Giotto referred in a painting to a wonderful comet he had seen in 1301. Could this have been Halley's comet? Why or why not?



Halley's comet is named for Edmond Halley (1656–1742), a British astronomer. He discovered the pattern of its visits and predicted its return in 1758—but didn't live to see the comet that bears his name.

10. **Ecology** Many animal species depend on other animals for food. For example, a fox's main source of nourishment is often rabbit. The fox is a *predator*, and the rabbit is its *prey*.

The populations over time of the foxes and rabbits in one forest might be modeled by the graphs below. Compare the two graphs.



- How often do the patterns repeat?
- How are the graphs alike? How are they different?
- Find the times when the fox population is changing fastest, and look at the rabbit population at those times. What do you notice?
- Challenge** Notice that when the fox population is dropping fastest, the rabbit population is at its lowest point.
 - What does a drop in the fox population do to the rabbit population? Why?
 - What would a low rabbit population mean for the foxes?
 - Use Parts i and ii to explain what is happening in the part of the graph where the fox population is dropping.



- 11. Earth Science** You may have heard of the El Niño weather effect. El Niño is a warming of the central and eastern Pacific Ocean. It affects weather in many countries, including the United States.

One climate measure is the Southern Oscillation Index (SOI), which is based on the difference in air pressure taken at the islands of Tahiti and in Darwin, Australia. When the index is negative, the West Coast of America usually experiences wet weather, and Australia usually experiences dry weather. When the index is positive, it tends to be drier in the Americas and wetter in Australia. Sustained negative values for the SOI accompany El Niño.

The table lists the approximate SOI measures every month from 1996 to 1998.

	1996	1997	1998
January	8.4	4.1	-23.5
February	1.1	13.3	-19.2
March	6.2	-8.5	-28.5
April	7.8	-16.2	-24.4
May	1.3	-22.4	0.5
June	13.9	-24.1	9.9
July	6.8	-9.5	14.6
August	4.6	-19.8	9.8
September	6.9	-14.8	11.1
October	4.2	-17.8	10.9
November	-0.1	-15.2	12.5
December	7.2	-9.1	13.3

Source: SOI archives, Australia Bureau of Meteorology, www.bom.gov.au

- Draw a graph of these data to show the pattern. Put the month on the horizontal axis.
- A significant El Niño effect hit in 1997 and 1998 when the SOI was negative for a long period. During that time, Australia suffered a drought. It was extremely wet in the Americas, causing massive mud slides in California, among other problems. Between what approximate dates (month and year) did this occur?

In your own words

What characteristics do the graphs of repeating patterns share? What do these kinds of graphs allow you to do with data points that may not be given?

12. A city has kept temperature records for over 100 years. From the records, someone calculated these statistics for the month of January:

Average daily temperature: 26°C

Percent of days that vary from the average:

- within 5°C of average: 60%
- within 10°C of average: 85%
- within 15°C of average: 100%

- a. Based on these long-term patterns for January, how many days would you expect the temperature to be from 21°C to 31°C?
- b. How many days would you expect the temperature to be from 16°C to 36°C?
- c. How many days would you expect the temperature to be above 36°C or below 16°C?
- d. Do you think the temperature in January could ever be 40°C? Explain your answer.

Mixed Review

Evaluate each expression for $k = 5$.

13. $\frac{1}{k} + \frac{k}{3}$

14. 1.2×10^k

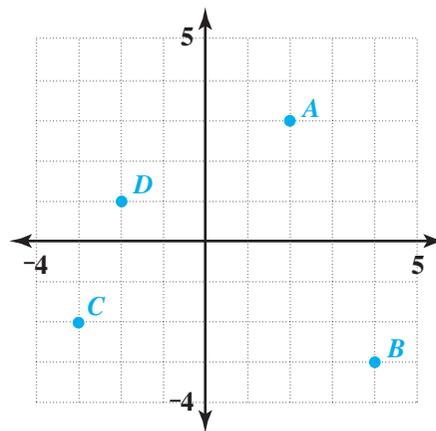
15. $\sqrt{k \cdot k}$

16. $3k^k$

17. $k^{2k} \times k^{-k}$

18. $\frac{4^k}{4^{k-2}}$

19. **Geometry** If you drew a quadrilateral using Points A–D as the vertices, what would the figure's perimeter be?



20. Rob has a free-throw average of 51%. In his next basketball game, he makes 4 out of 7 free-throws he attempts. Did he do better or worse than his average? Explain.



- 21. Social Studies** The landlocked South American country of Paraguay is divided into 17 regions called *departments*. The table lists the area and population of each department.

Department	Area (km ²)	Population (2002 census)
Alto Paraguay	82,349	15,008
Alto Paraná	14,895	563,042
Amambay	12,933	113,888
Boquerón	91,669	45,617
Caaguazú	11,474	448,983
Caazapá	9,496	139,241
Canendiyú	14,667	140,551
Central	2,465	1,363,399
Concepción	18,051	180,277
Cordillera	4,948	234,805
Guairá	3,846	176,933
Itapúa	16,525	463,410
Misiones	9,556	103,633
Ñeembucú	12,147	76,738
Paraguarí	8,705	226,514
Presidente Hayes	72,907	81,876
San Pedro	20,002	318,787

Source: www.citypopulation.de

- Which department has the greatest population density (people per square kilometer)? Which has the least?
- How many times greater is the department with the most area than the department with the least?
- Which department has the median area? What is that area?
- Which department has the median population? What is that population?

- 22.** The Paraguay currency unit is the *guaraní*. In June 2003, 1 U.S. dollar was worth about 6,119 guaraní.

- How many guaraní would you have received in exchange for 40 U.S. dollars?
- How many U.S. dollars would you have received in exchange for 1,000,000 guaraní?



VOCABULARY

line graph
multiplicative
inverse

MATERIALS

graph paper

Chapter Summary

In this chapter, you worked with several types of graphs. First you looked at graphs that show how one variable changes over time. Then you saw how relationships with a square, with an inverse, or with a variable as an exponent look when graphed.

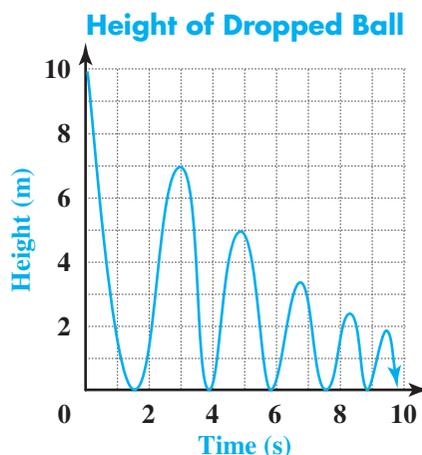
You used graphs to solve problems for different situations. You also saw situations—including some related to weather—that involve repeating patterns. Some graphs, such as those for inverse relationships and bank-account balances, had points that could not be connected.

Strategies and Applications

The questions in this section will help you review and apply the important ideas and strategies developed in this chapter.

Interpreting graphs over time

- Luis's birthday party began around 2:00 P.M. Kasinda noticed that the noise level changed as the party progressed. Use the following events to sketch a graph that may show how the noise level rose and fell over time.
 - Most of the guests arrived between 2:00 P.M. and 2:35 P.M.
 - The guests sang "Happy Birthday" to Luis at about 2:55 P.M.
 - Luis blew out the candles on his cake at about 2:58 P.M.
 - Everyone applauded after Luis blew out the candles.
 - At 3:30 P.M., Kasinda turned up the music so people could dance.
- Kenyon dropped a ball from a height of about 10 meters. Lucetta carefully measured the maximum heights of the ball as it bounced, and Kenyon timed when it reached each maximum height. Using what they know about the way dropped objects fall, they created this graph.



- a. Explain why the graph looks as it does.
- b. About what height does the ball reach on its first bounce?
- c. About how long does the second bounce last? That is, how much time passes between the second time the ball hits the ground and the third time?

Recognizing graphs of square, inverse, and exponential relationships

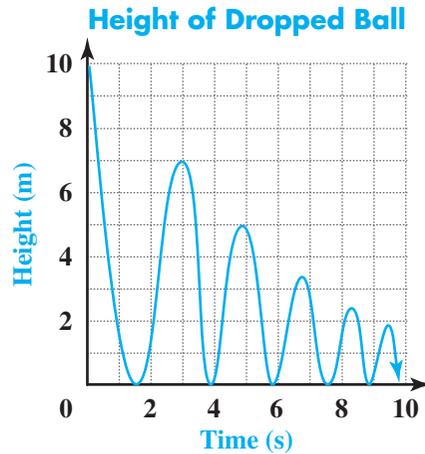
- 3. Think about the similarities and the differences among the formulas in the box.

$s = 9.8t$	the speed s in meters per second of a falling body after t seconds
$A = \pi r^2$	the area A of a circle with radius r
$d = 4.9t^2$	the distance d in meters a falling body has fallen after t seconds
$B = 100(1.04^n)$	the balance B in a bank account after n years if \$100 is left in the account and the account pays 4% interest compounded annually
$W = \frac{36}{L}$	the width W of a rectangle with length L and area 36 square units
$S = 6L^2$	the surface area S of a cube with edge length L
$B = \frac{1}{4}F$	the amount B of butter needed in a recipe for F units of flour
$b = 3(4^t)$	the number of bacteria b in a petri dish after t hours, if the dish held 3 bacteria to begin and they quadruple every hour
$Q = 150 - 10t$	the quantity Q of water in liters remaining in a tank after t minutes, if the tank holds 150 liters and is drained at a rate of 10 liters per minute
$V = \frac{500}{P}$	the volume V of a certain amount of gas at a certain temperature as pressure P varies

- a. Organize the formulas into four groups:
 - i. those you think have graphs that are straight lines
 - ii. those you think have graphs most like the graph of $y = x^2$
 - iii. those you think have graphs most like the graph of $y = \frac{1}{x}$
 - iv. those you think have graphs most like the graph of $y = 2^x$
- b. Choose four formulas, one from each category in Part a. Find several pairs of values, and draw a graph for each formula.

Using graphs to solve problems and equations

4. The graph shows how the height of a dropped ball changed over time.



- a. When did the ball have a height of 6 meters?
- b. When was the last time the ball reached a height of 4.9 meters?

Recognizing and using repeating patterns in graphs

5. Although a large portion of Arizona is warm or even hot year-round, some areas have a more varied climate. The table shows the average minimum temperatures in Flagstaff, Arizona, for three years.

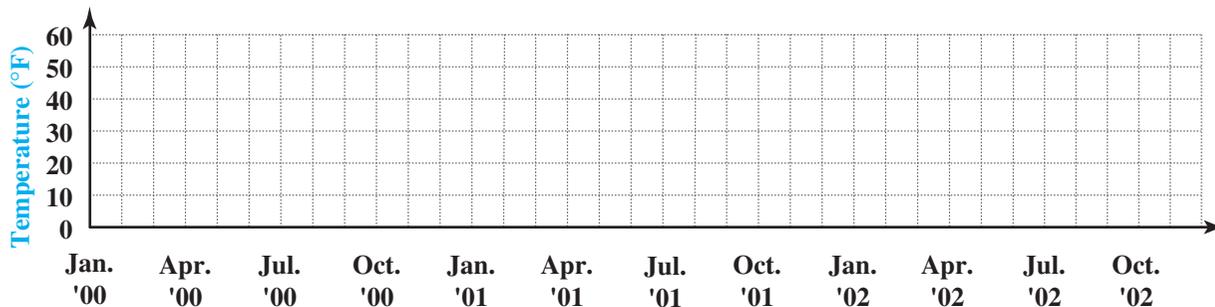


Month	Average Minimum Temperature (°F)		
	2000	2001	2002
Jan.	18.7	15.0	17.9
Feb.	22.9	17.0	17.3
Mar.	22.6	24.7	20.0
Apr.	28.7	29.9	31.4
May	37.0	37.4	33.1
Jun.	44.5	43.1	42.8
Jul.	49.1	50.3	55.5
Aug.	51.0	49.5	49.7
Sep.	42.9	41.3	43.2
Oct.	32.6	33.2	31.5
Nov.	17.7	24.7	25.2
Dec.	20.5	12.9	15.4

Source: www.wrcc.dri.edu

- a. Plot the Flagstaff, Arizona, temperature data on axes like those shown below. Connect the points using line segments.

Average Minimum Temperature



- b. Describe the pattern in your graph.



Demonstrating Skills

Draw a rough sketch to show how the graph of each equation will look.

6. $y = \frac{3}{x}$

7. $y = -3x + 5$

8. $y = 3^x$

9. $y = x^2 + 5$