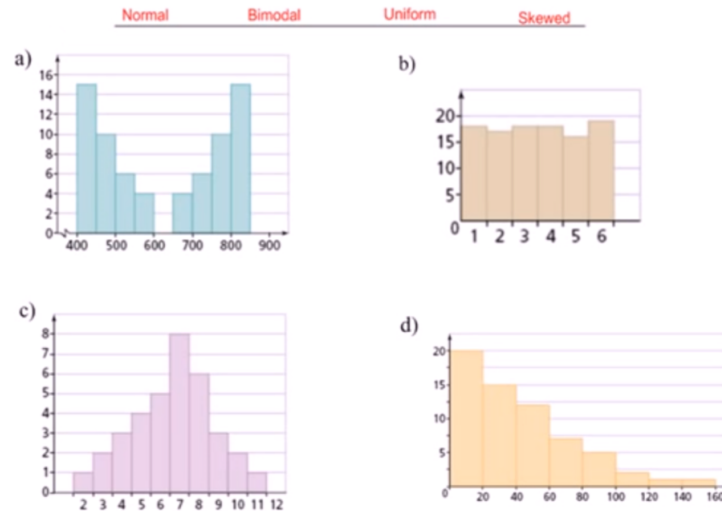


Match these Distributions types with their graphs

Know Your Distribution Types



2. Two machines, A and B, fill empty packets with soap powder. A sample of boxes was taken from each machine and the weight of powder (in kg) was recorded.

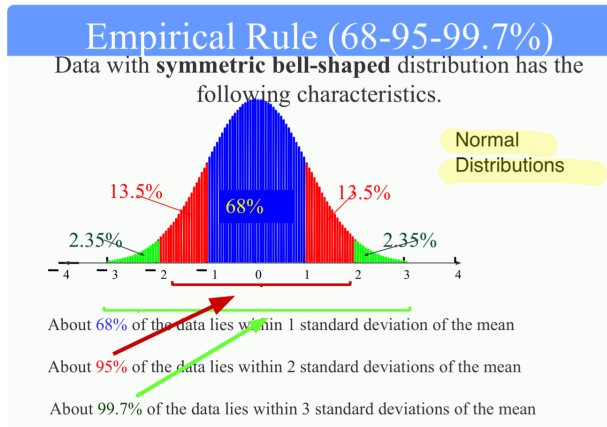
<i>A</i>	2.27	2.31	2.18	2.2	2.26	2.24			
<i>B</i>	2.78	2.62	2.61	2.51	2.59	2.67	2.62	2.68	2.70

- (a) Find the mean and standard deviation for each machine.
- (b) Which machine is most consistent?
3. Two groups of students were trying to find the acceleration due to gravity. Each group conducted 5 experiments.

<i>Group A</i>	9.4	9.6	10.2	10.8	10.1
<i>Group B</i>	9.5	9.7	9.6	9.4	9.8

Find the mean and standard deviation for each group, and comment on their results.

Normal Distributions Mean = Median = Mode



①

The lifespans of zebras in a particular zoo are normally distributed. The average zebra lives 20.5 years; the standard deviation is 3.9 years.

Use the empirical rule (68 – 95 – 99.7%) to estimate the probability of a zebra living between 16.6 and 24.4 years.

②

The lifespans of zebras in a particular zoo are normally distributed. The average zebra lives 20.5 years; the standard deviation is 3.9 years.

Use the empirical rule (68 – 95 – 99.7%) to estimate the probability of a zebra living less than 32.2 years.

③

The lifespans of gorillas in a particular zoo are normally distributed. The average gorilla lives 16 years; the standard deviation is 1.7 years.

Use the empirical rule (68 – 95 – 99.7%) to estimate the probability of a gorilla living longer than 14.3 years.

④

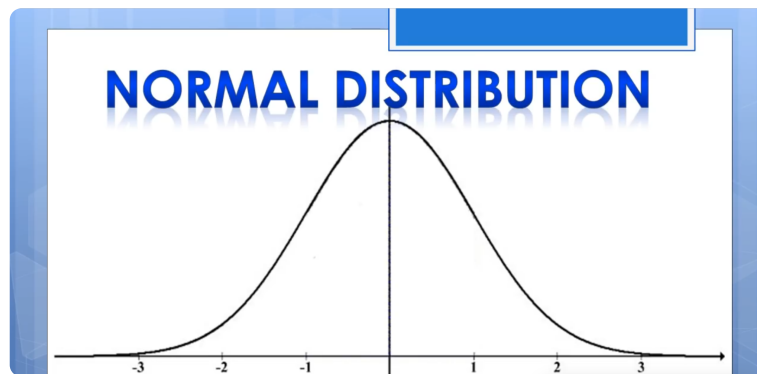
The lifespans of tigers in a particular zoo are normally distributed. The average tiger lives 22.4 years; the standard deviation is 2.7 years.

Use the empirical rule (68 – 95 – 99.7%) to estimate the probability of a tiger living between 27.8 and 30.5 years.

Empirical Rule/Chebyshev's Theorem Worksheet

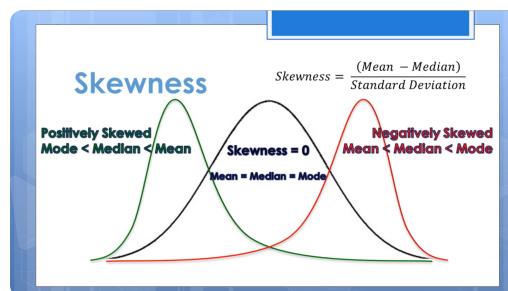
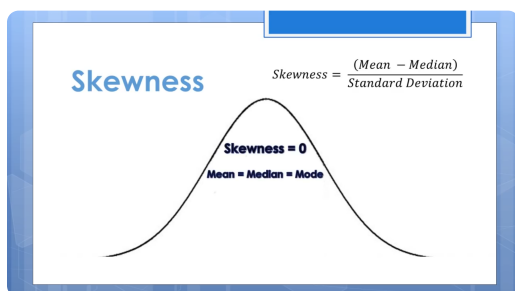
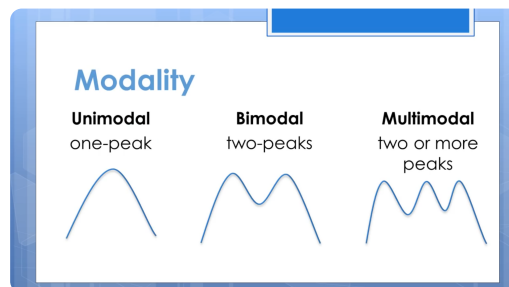
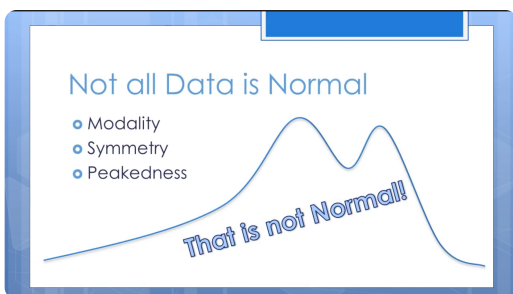
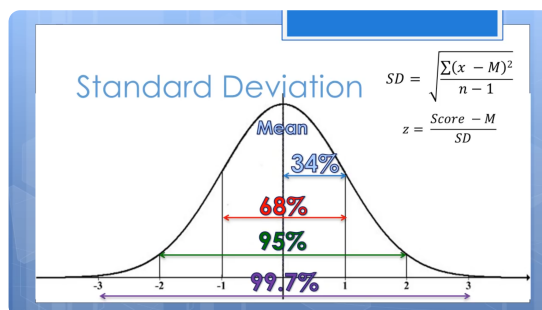
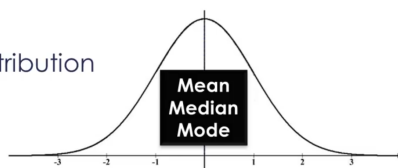
- 1) Adult IQ scores have a bell-shaped distribution with a mean of 100 and a standard deviation of 15. Use the Empirical Rule to find the percentage of adults with scores between 70 and 130.
- 2) Lengths of pregnancies of humans are normally distributed with a mean of 265 days and a standard deviation of 10 days. Use the Empirical Rule to determine the percentage of women whose pregnancies are between 255 and 275 days.
- 3) A competency test has scores with a mean of 82 and a standard deviation of 2. A histogram of the data shows that the distribution is normal. Between what two values do about 99.7% of the values lie?
- 4) A placement exam for entrance into a math class yields a mean of 80 and a standard deviation of 10. The distribution of the scores is roughly bell-shaped. Use the Empirical Rule to find the percentage of scores that lie between 60 and 80.
- 5) SAT verbal scores are normally distributed with a mean of 433 and a standard deviation of 90. Use the Empirical Rule to determine what percent of the scores lie between 433 and 523.
- 6) The average IQ of students in a particular calculus class is 110, with a standard deviation of 5. The distribution is roughly bell-shaped. Use the Empirical Rule to find the percentage of students with an IQ above 120.
- 7) At a tennis tournament a statistician keeps track of every serve. The statistician reported that the mean serve speed of a particular player was 97 miles per hour (mph) and the standard deviation of the serve speeds was 10 mph. If nothing is known about the shape of the distribution, give an interval that will contain the speeds of at least three-fourths of the player's serves.
- 8) Heights of adult women have a mean of 63.6 in. and a standard deviation of 2.5 in. What does Chebyshev's Theorem say about the percentage of women with heights between 56.1 in. and 71.1 in.?
- 9) A study was designed to investigate the effects of two variables - (1) a student's level of mathematical anxiety and (2) teaching method - on a student's achievement in a mathematics course. Students who had a low level of mathematical anxiety were taught using the traditional expository method. These students obtained a mean score of 250 with a standard deviation of 50 on a standardized test. Assuming no information concerning the shape of the distribution is known, what percentage of the students scored between 150 and 350?





Key Features

- Centered
- Fixed score distribution
- Unimodal
- Symmetrical



How to do normal distribution problems on the calculator:

EXAMPLE 1

On a standardized test, the test scores are normally distributed with a mean of 60 and a standard deviation of 6.

- a. Of the data, 84% of the scores are at or below what score?
- b. Of the data, 16% of the scores are at or below what score?
- c. What is the z-score of a score of 48?
- d. If 2,000 students took the test, how many would be expected to score at or below 48?

$$\text{Zscore} = \frac{X - \bar{X}}{\sigma}$$

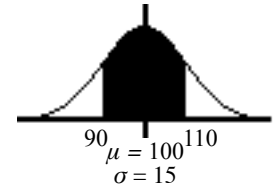
Using Your TI-83/84 Calculator: Normal Probability Distributions Elementary Statistics

Always start by drawing a sketch of the normal distribution that you are working with. Shade in the area (probability) that you are given or trying to find, and label the mean, standard deviation, lower bound, and upper bound that you are given or trying to find. Don't worry about making your drawing to scale; the purpose of the sketch is to get you thinking clearly about the problem you are trying to solve. For illustration purposes, let's consider the distribution of adult scores on the Weschler IQ test. These IQ scores are normally distributed with $\mu = 100$ and $\sigma = 15$.

Using the normalcdf command ← 2nd VARS 2:normalcdf(

The normalcdf command is used for finding an area under the normal density curve. This area corresponds to the probability of randomly selecting a value between the specified lower and upper bounds. You can also interpret this area as the percentage of all values that fall between the two specified boundaries.

- Let's find the percentage of adults who score between 90 and 110 on the Weschler IQ test. Begin by sketching the distribution and labeling the relevant information. We are ultimately trying to find the area under the normal density curve that is bounded by 90 and 110, so shade in that area on your sketch.



- Press **2nd**[VARS] for the [DISTR] menu. Scroll down to 2:normalcdf(and then press **ENTER**.
- Enter the two x values (or z scores) that form the boundaries of the area that you are trying to find, the population mean, and the population standard deviation using the following syntax: lowerbound, upperbound, μ , σ and then press **ENTER**. For this example, type 90, 110, 100, 15) and then press **ENTER**.

```

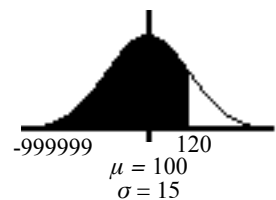
DISTR DRAW
1:normalPdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tPdf(
6:tCdf(
7:χ²Pdf(
    
```

```

normalcdf(90,110
,100,15)
.495015066
    
```

- Your calculator will return the area under the normal curve bounded by 90 and 110. Thus, we find that 49.5% of adults score between 90 and 110 on the Weschler IQ test. Remember to round percentages to three significant figures.

- What is the probability that a randomly selected adult scores less than 120 on the Weschler IQ test? Problems like this are a bit trickier because your calculator requires both a lower bound and an upper bound. Ultimately, you need to specify an approximation of $-\infty$ for the lower bound. In most cases, -999999 is a good choice for the lower bound. Hence, the syntax for problems of this sort is normalcdf(-999999, upperbound, μ , σ).

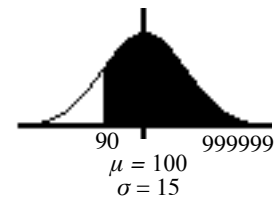


- Start by drawing a sketch. Then, press **2nd**[VARS] for the [DISTR] menu. Scroll down to 2:normalcdf(and press **ENTER**. Type **(-)**999999, 120, 100, 15) and then press **ENTER** again. We find that the probability of randomly selecting an adult whose IQ is below 120 is 0.909. In symbols, $P(x \leq 120) = 0.909$. Remember to round probability values to 3 significant figures.

```

normalcdf(-99999
9,120,100,15)
.9087887181
    
```

- What percentage of adults score at least 90 on the Weschler IQ test? Because your calculator requires both a lower bound and an upper bound, you will need to specify an approximation of ∞ for your upper bound. Using 999999 is good enough in most cases. Hence, the syntax for problems of this sort is `normalcdf(lowerbound, 999999, μ , σ)`.
- Start by drawing a sketch. Then, press `2nd|VARS` for the `[DISTR]` menu. Scroll down to `2:normalcdf(` and press `[ENTER]`. Type `90, 999999, 100, 15)` and then press `[ENTER]` again.
- We find that 74.8% of adults score at least 90 on the Weschler IQ test. Remember to round to 3 significant figures.



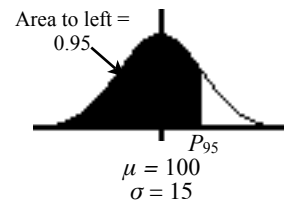
```
normalcdf(90, 999
999, 100, 15)
.747507533
```

Using the `invNorm` command `2nd VARS 3:invNorm(`

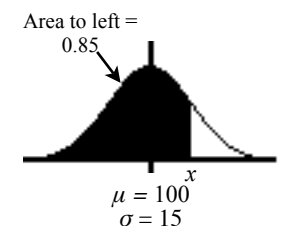
Use the `invNorm` command when you are given a probability or percentage and asked to find an x value or z score. This command is often used to find values corresponding to percentiles or quartiles. Your calculator requires that you enter the cumulative area to the left of the desired value; drawing a sketch is very useful for making sure you enter the correct area. Sometimes you will need to work with an area other than the one specified by the problem.

- Press `2nd|VARS` for the `[DISTR]` menu. Scroll down to `3:invNorm(` and press `[ENTER]`.
- Let's start by finding the IQ score corresponding to the 95th percentile (P_{95}). Begin by drawing a sketch and labeling it.
- Enter the total area to the left of the desired x value (or z score) using the following syntax: `invNorm(area to left, μ , σ)` and then press `[ENTER]`. For this example, type `95, 100, 15)` and then press `[ENTER]`.
- Your calculator will return the x value (or z score). For this example, we find that the Weschler IQ score corresponding to the 95th percentile is 124.7. Round your answer to one more decimal place than what was provided for μ .
- Let's try a trickier example. This time, find the IQ score separating the top 15% of all Weschler IQ scores from the rest. Start by drawing a sketch. Given that the total area under the normal density curve is always 1, the area to the left of the IQ score that we are seeking can be found by subtracting the given area from 1. In this case, $1 - 0.15 = 0.85$. Don't forget to convert percentages to their equivalent decimal values.
- Press `2nd|VARS` for the `[DISTR]` menu. Scroll down to `3:invNorm(` and press `[ENTER]`.
- The area to the left of the IQ score we are seeking is 0.85, so type `0.85, 100, 15)` and then press `[ENTER]`. We find that an IQ score of 115.5 separates the top 15% of all adult Weschler IQ scores from the rest.

```
0:1: DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7:χ²pdf(
```



```
invNorm(0.95, 100
, 15)
124.6728044
```

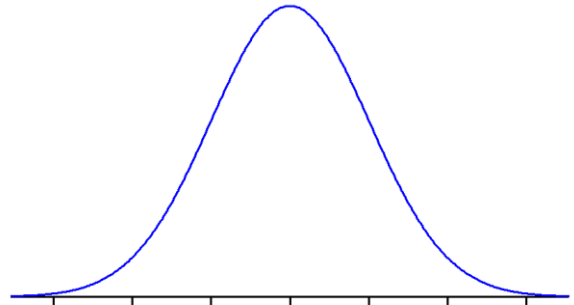


```
invNorm(0.85, 100
, 15)
115.5465007
```

Practice Exercises: Normal Distributions

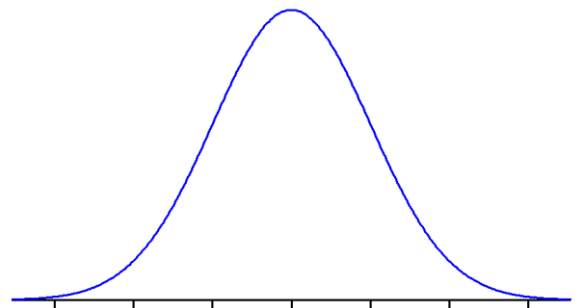
1. The distribution of blood cholesterol levels in 14-year-old boys is roughly normal; the mean is 165 milligrams of cholesterol per deciliter of blood and the standard deviation is 30.

- a) Label the horizontal axis appropriately on the Normal curve at right.
- b) What proportion of 14-year-old boys have level between 120 and 200? Shade the appropriate area in the figure, and then determine the proportion.



c) What proportion have level *over* 20?

d) [5] What level separates the boys who have level in the highest 70% from those in the lowest 30%?



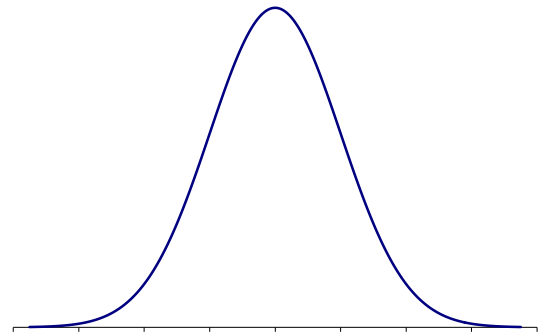
2. Consider SAT scores for 2009 high school graduates. The scores are approximately Normally distributed.

- a) What is the approximate percent of graduates scoring within one standard deviation of the mean score? _____ %
- b) Jeff's Z score is -1.75 . What does this tell us about Jeff's SAT score?
 Jeff's score is _____ standard deviations _____ the mean.
- c) All the SAT scores are converted to Z scores. What are the mean and standard deviation of these Z scores? (Visualize the Z scale beneath the SAT scale.)
 Mean of Z scores = _____ S D of Z scores = _____

Practice Exercises: Normal Distributions

3. Lifetimes of a certain brand of tires is approximately normally distributed with mean 42,500 miles and standard deviation 3,200 miles

a) Fill in appropriate axis label at the tick marks on the graph at right. Identify the areas associated with the 68-95-99.7 rule.



b) In what range do the middle 95% of tire lifetimes lie?

_____ to _____

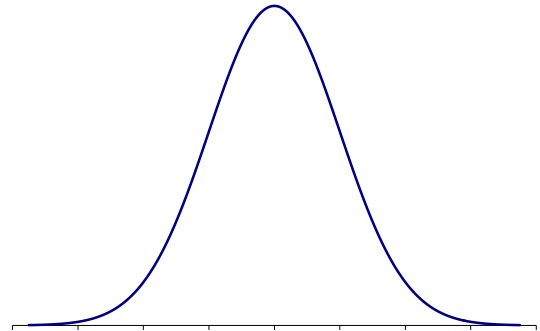
c) What proportion of tires last over 45,000 miles?

_____ nearest 0.0001

d) What is the median lifetime? _____

e) How far do the longest-lasting 4% of these tires roll? (What percentile are you finding here?)

_____ miles



f) Customers get a refund if a tire has unacceptable tread that puts it in the bottom 12% of all tires. At what mileage is a customer no longer entitled to a refund?

(What percentile are you finding here?)

_____ miles

More Exercises

1. The angiogram is a standard diagnostic test used in clinical medicine to detect stroke in patients. This test has some risks for the patient, and several noninvasive techniques have been developed that are hoped to be as effective as the angiogram. One such method utilizes that measurement of cerebral blood flow (CBF) in the brain, since stroke patients tend to have lower levels of CBF than normal. Among healthy people, CBF is normally distributed with mean 75 and standard deviation 17. Patients are classified as being at risk for stroke if their CBF is below 40. What proportion of normal patients will be mistakenly classified as being at risk for stroke?
2. Maple tree diameters in a forest area are normally distributed with mean 10 inches and standard deviation 2.2 inches. Find the proportion of trees having a diameter greater than 15 inches.
3. Our subjects are 35-44-year-old males whose blood pressures are normally distributed with mean 80 and standard deviation 12. A borderline hypertensive is defined as a person whose diastolic blood pressure is between 90 and 95 mm Hg inclusive; what proportion of subjects are borderline hypertensive? A hypertensive is a person whose diastolic blood pressure is above 95 mm Hg; what proportion of subjects are hypertensive?
4. White blood cell (WBC) count per cubic millimeter of whole blood has approximately the Normal distribution with mean 7500 and standard deviation 1750. The lowest 2% of all WBC counts are defined to be probable risks. How low must one's WBC count be to fall in the at-risk group?
5. The resting heart rate for healthy adult horses averages 46 beats per minute with a standard deviation of 8 beats per minute. A horse whose resting heart rate is in the upper 10% of the distribution of heart rates may have a secondary infection or illness that needs to be treated. How fast must a healthy horse's heart be beating to fall into this at-risk group?

Applying Skills

In 11–14, select the numeral that precedes the choice that best completes the statement or answers the question.

11. The playing life of a Euclid mp3 player is normally distributed with a mean of 30,000 hours and a standard deviation of 500 hours. Matt's mp3 player lasted for 31,500 hours. His mp3 player lasted longer than what percent of other Euclid mp3 players?
(1) 68% (2) 95% (3) 99.7% (4) more than 99.8%
12. The scores of a test are normally distributed. If the mean is 50 and the standard deviation is 8, then a student who scored 38 had a z-score of
(1) 1.5 (2) -1.5 (3) 12 (4) -12
13. The heights of 10-year-old children are normally distributed with a mean of 138 centimeters with a standard deviation of 5 centimeters. The height of a 10-year-old child who is as tall as or taller than 95.6% of all 10-year-old children is
(1) between 138 and 140 cm. (2) between 140 and 145 cm.
(3) between 145 and 148 cm. (4) taller than 148 cm.
14. The heights of 200 women are normally distributed. The mean height is 170 centimeters with a standard deviation of 10 centimeters. What is the best estimate of the number of women in this group who are between 160 and 170 centimeters tall?
(1) 20 (2) 34 (3) 68 (4) 136
15. When coffee is packed by machine into 16-ounce cans, the amount can vary. The mean weight is 16.1 ounces and the standard deviation is 0.04 ounce. The weight of the coffee approximates a normal distribution.
 - a. What percent of the cans of coffee can be expected to contain less than 16 ounces of coffee?
 - b. What percent of the cans of coffee can be expected to contain between 16.0 and 16.2 ounces of coffee?

- 16.** The length of time that it takes Ken to drive to work represents a normal distribution with a mean of 25 minutes and a standard deviation of 4.5 minutes. If Ken allows 35 minutes to get to work, what percent of the time can he expect to be late?
- 17.** A librarian estimates that the average number of books checked out by a library patron is 4 with a standard deviation of 2 books. If the number of books checked out each day approximates a normal distribution, what percent of the library patrons checked out more than 7 books yesterday?
- 18.** The heights of a group of women are normally distributed with a mean of 170 centimeters and a standard deviation of 10 centimeters. What is the z-score of a member of the group who is 165 centimeters tall?
- 19.** The test grades for a standardized test are normally distributed with a mean of 50. A grade of 60 represents a z-score of 1.25. What is the standard deviation of the data?
- 20.** Nora scored 88 on a math test that had a mean of 80 and a standard deviation of 5. She also scored 80 on a science test that had a mean of 70 and a standard deviation of 3. On which test did Nora perform better compared with other students who took the tests?